Chapter 13, Solution 1.

Given: Weight of satellite, $W = 1000 \text{ lb}$

Speed of satellite, $v = 14,000 \text{ mi/h}$

Find: Kinetic energy, T

$$v = (14,000 \text{ mi/h})(5280 \text{ ft/mi})\left(\frac{\text{h}}{3600 \text{ s}}\right) = 20,533 \text{ ft/s}$$

$$\text{Mass of satellite} = \frac{(1000 \text{ lb})}{\left(32.2 \text{ ft/s}^2\right)} = 31.0559 \text{ lb s}^2\text{/ft}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}(31.0559)(20,533)^2 = 6.5466 \times 10^9 \text{ lb} \cdot \text{ft}$$

$$T = 6.55 \times 10^9 \text{ lb} \cdot \text{ft}$$

Note: Acceleration of gravity has no effect on the mass of the satellite.

$$T = 6.55 \times 10^9 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

Chapter 13, Solution 2.

$$v = \left(\frac{\text{Circumference}}{\text{Time}}\right)$$

$$v = \frac{(2\pi)(6370\text{ km} + 35{,}800\text{ km})(1000\text{ m/km})}{(23\text{ hr})(3600\text{ s/hr}) + (56\text{ min})(60\text{ s/hr})} = 3075.2\text{ m/s}$$

$$v = 3075.2\text{ m/s}$$

Kinetic energy, $T = \frac{1}{2}mv^2 = \frac{1}{2}(500\text{ kg})(3075.2\text{ m/s})^2$

$T = 2.36$ GJ ◀

Chapter 13, Solution 3.

Given: Mass of stone, $m = 2\text{ kg}$

Velocity of stone, $v = 24\text{ m/s}$

Acceleration of gravity on the moon, $g_m = 1.62\text{ m/s}^2$

Find:

(*a*) Kinetic energy, *T*

Height *h*, from which the stone was dropped

(*b*) *T* and *h* on the Moon

(*a*) On the Earth $T = \frac{1}{2}mv^2 = \frac{1}{2}(2\text{ kg})(24\text{ m/s})^2 = 576\text{ N}\cdot\text{m}$ $T = 576\text{ J}$ ◀

$$W = mg = (2\text{ kg})\left(9.81\text{ m/s}^2\right) = 19.62\text{ N}$$

$$T_1 = U_{1-2} = T_2 \qquad T_1 = 0 \qquad U_{1-2} = Wh \qquad T_2 = 576\text{ J}$$

$$Wh = T_2 \qquad h = \frac{T_2}{W} = \frac{(576\text{ N}\cdot\text{m})}{(19.62\text{ N})} = 29.36\text{ m}$$

$h = 29.4\text{ m}$ ◀

(*b*) On the Moon

Mass is unchanged. $m = 2\text{ kg}$

Thus *T* is unchanged. $T = 576\text{ J}$ ◀

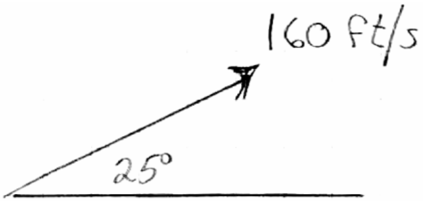
Weight on the moon is, $W_m = mg_m = (2\text{ kg})\left(1.62\text{ m/s}^2\right)$

$$W_m = 3.24\text{ N}$$

$$h_m = \frac{T}{W_m} = \frac{(576\text{ N}\cdot\text{m})}{3.24\text{ N}} = 177.8\text{ m}$$

$h_m = 177.8\text{ m}$ ◀

Chapter 13, Solution 4.



$$m = 1.6203\left(\frac{1\text{ lb}}{16\text{ oz}}\right)\left(\frac{1}{32.2\text{ ft/s}^2}\right)$$

$$(a)\ T = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1.62}{16(32.2)}\right)(160\text{ ft/s})^2$$

$T = 40.2$ ft-lb ◀

At maximum height,

$$v = v_x = (160\text{ ft/s})\cos 25°$$

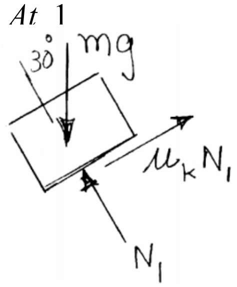
$$(b)\ T = \frac{1}{2}m\,(160\cos 25°)^2$$

$T = 33.1$ ft-lb ◀

Chapter 13, Solution 5.

Use work and energy with position 1 at *A* and position 2 at *C*.

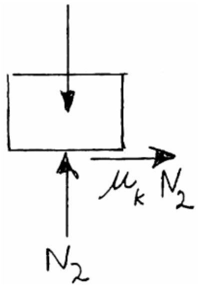
At 1



$$+\!\uparrow \Sigma F_y = 0 \quad \Rightarrow N_1 - mg\cos 30° = 0$$

$$N_1 = mg\cos 30°$$

At 2



$$+\!\uparrow \Sigma F_y = 0 \quad \Rightarrow N_2 - mg = 0$$

$$N_2 = mg$$

Work and energy

$$T_1 + V_{1\to 2} = T_2 \qquad (1)$$

Where

$$T_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}m(4\text{ ft/s})^2 = 8m$$

$$V_{1\to 2} = -\mu_k N_1 d - \mu_k N_2(20) + mg\,(d\sin 30°)$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(8)^2 = 32m$$

Into (1)

$$8\cancel{m} - \mu_k \cancel{m}gd\cos 30° - \mu_k \cancel{m}g(20) + \cancel{m}gd\sin 30° = 32\cancel{m}$$

Solve for $d = \dfrac{32 - 8 + \mu_k g\,20}{-\mu_k g\cos 30° + g\sin 30°} = \dfrac{32 - 8 + (0.25)(32.2)(20)}{(-0.25)\,(32.2)(0.866 + 32.2(0.5))} = 20.3\text{ ft}$ ◀

Chapter 13, Solution 6.

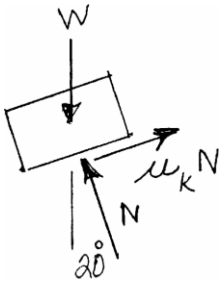
(*a*) Use work and energy from *A* to *B*. $T_1 + V_{1\to 2} = T_2$

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)(40)^2 = 1242.24 \text{ ft}\cdot\text{lb}$$

$$T_2 = 0 \quad \text{(Stops at top)}$$

$$U_{1\to 2} = -\mu N x - mg\,x\sin 20°$$

N is needed



$$+\uparrow\ \Sigma F_y = 0 \qquad \Rightarrow N = W\cos 20° = (50 \text{ lb})\cos 20° = 46.985 \text{ lb}$$

So

$$U_{1\to 2} = -0.15(46.985)x - 50\sin 20° x$$
$$= -24.149x$$

Substitute

$$1242.24 - 24.149x = 0$$

$$x = 51.44 \text{ ft}$$

$x = 51.4$ ft ◀

(*b*) Package returns to *A* – use work and energy from *B* to *A*

$$T_2 + U_{2\to 3} = T_3$$

Where $T_2 = 0$ (At *B*)

$$U_{2\to 3} = W\sin 20° x - \mu_k N x$$
$$= (50)\sin 20°(51.44) - 0.15(46.985)(51.44)$$
$$= 517.13 \text{ ft}\cdot\text{lb}$$

$$T_3 = \frac{1}{2}mv_3^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)v_3^2 = 0.7764\,v_3^2$$

Substitute

$$0 + 517.13 = 0.7764\,v_3^2$$

$$v_3 = 25.81 \text{ ft/s}$$

$v_3 = 25.8$ ft/s ↙ 20° ◀

(c) Energy dissipated is equal to change of kinetic energy

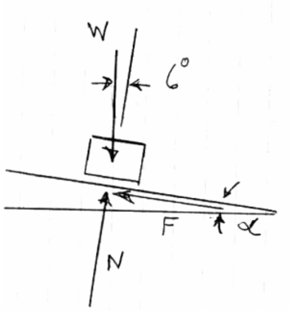
$$T_1 - T_3 = \frac{1}{2}\,m\,v_1^2 - \frac{1}{2}\,m\,v_2^2$$

$$= \frac{1}{2}\left(\frac{50}{32.2}\right)(40^2 - 25.81^2)$$

$$= 725 \text{ ft}\cdot\text{lb}$$

Energy dissipated $= 725$ ft·lb ◀

Chapter 13, Solution 7.



Given: Automobile Weight $W = mg = (2000 \text{ kg})(9.81)$

$W = 19{,}620 \text{ N}$

Initial Velocity A, $v_A = 0 \text{ m/s}$

Incline Angle, $\alpha = 6°$

Vehicle brakes at impending slip for 20 m from B to C

$v_C = 0$

Find; speed of automobile at point B, v_B

Coefficient of static friction, μ

(*a*)

$$U_{A\to B} = Wh_{A\to B} - (19620 \text{ N})(150 \text{ m})\sin 6°$$

$$= 307.63\times 10^3 \text{ N}\cdot\text{m}$$

$$U_{A\to B} = T_B - T_A = \frac{1}{2}mv^2 - 0$$

$$307.63\times 10^3 \text{ N}\cdot\text{m} = \frac{1}{2}(2000\,\text{kg})v_B^2 - 0$$

$v_B = 17.54 \text{ m/s}$ ◀

(*b*)

$$U_{A\to C} = Wh_{A\to C} - Fd_{B\to C} = T_C - T_A = 0$$

$$d_{B\to C} = 20 \text{ m} \qquad F = \mu \text{ N}$$

Where μ = coefficient of static friction

$$U_{A\to C} = (19620\,\text{N})(\sin 6°)(170\,\text{m}) - F(20\,\text{m})$$

$$F = \mu(19620\,\text{N})\cos 6°$$

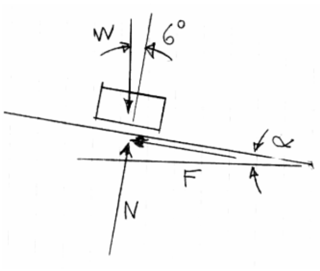
$$(19620\,\text{N})(\sin 6°)(170\,\text{m}) - \mu(19620\,\text{N})(\cos 6°)(20\,\text{m}) = 0$$

$$\mu = \frac{170}{20}\tan 6° = 0.893$$

$\mu = 0.893$ ◀

Chapter 13, Solution 8.

Given: Automobile weight, $W = (2000)(9.81) = 19620\text{ N}$



Initial velocity at A, $v_A = 0\text{ m/s}$

Incline Angle, $\alpha = 6°$

Vehicle costs 150 m from A to B

Vehicle skids 20 m from B to C

Dynamic friction coefficient, $\mu = 0.75$

Find: Work done on automobile by air resistance and rolling resistance between points A and C.

$$U_{A\to C} = U_R + Wh_{A\to C} - F\,(20\text{ m}) = T_C - T_A = 0$$

$$F = \mu\text{N} = 0.75\ (19620\text{ N})\cos 6°$$

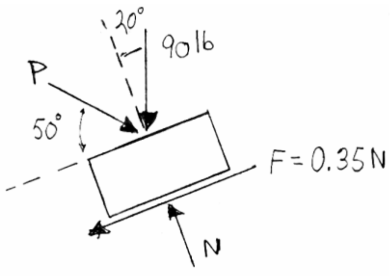
$$U_R = \text{Resistance work}$$

$$= 0.75\ (19620\text{ N})\cos 6°(20\text{ m}) - (19620\text{ N})\sin 6°(170\text{ m})$$

$$U_R = -56.0\times 10^3\text{ N}\cdot\text{m}$$

$$or\ -56.0\times 10^3\text{J} ◀$$

Chapter 13, Solution 9.



$$\sum F_N = N - 90\cos 20° - P\sin 50° = 0$$

$$N = 90\cos 20° + P\sin 50°$$

$$U_{1\to 2} = [P\cos 50° - 90\sin 20° - 0.35\ \text{N}](3\ \text{ft})$$

$$T_2 = \frac{1}{2}\left(\frac{90\ \text{lb}}{32.2\ \text{ft/s}^2}\right)(2\ \text{ft/s})^2$$

$$(P\ \cos 50°)\ 3 - (90\sin 20°)\ (3) - 0.35\ (90\cos 20° + P\sin 50°)3 = T_2$$

$$P\ (3\ \cos 50° - 0.35(3)\ \sin 50°) = 90\sin 20°(3) + 0.35\ (90\cos 20°)(3) + \frac{1}{2}\left(\frac{90}{32.2}\right)(2)^2$$

$$P = \frac{186.736}{1.12402} = 166.1\ \text{lb} ◀$$

Chapter 13, Solution 10.

(*a*) First stage: $U_{1\to 2} = (5.5 - 3)(50) = 125\ \text{lb}\cdot\text{ft} = T_2$

$$T_2 = \frac{1}{2}\left(\frac{3}{32.2}\right)v_2^2$$

$v_2 = 51.8\ \text{ft/s}$ ◀

(*b*) At the top: $U_{2\to 3} = -3(h - 50) = 0 - 125$

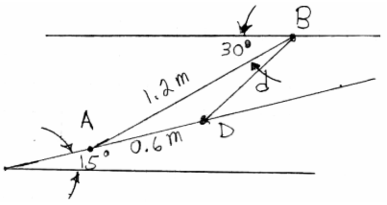
$$\therefore h = \frac{275}{3},$$

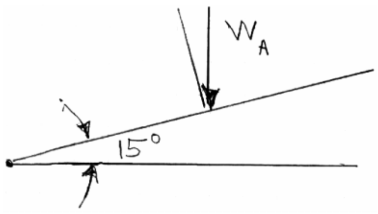
$h = 91.7\ \text{ft}$ ◀

(*c*) At the return: $U_{3\to 4} = +3(91.6667) = T_4 = \frac{1}{2}\left(\frac{3}{32.2}\right)v_4^2$

$v_4 = 76.8\ \text{ft/s}$ ◀

Chapter 13, Solution 11.





$W_A = 7(9.81) = 68.67$ N

Given: Block A is released from rest and moves up incline 0.6 m.

Friction and other masses are neglected

Find: Velocity of the block after 0.6 m, v

From the Law of Cosines

$$d^2 = (1.2)^2 + (0.6)^2 - 2(1.2)(0.6)\cos 15°$$

$$d^2 = 0.4091\text{ m}^2$$

$$d = 0.63958\text{ m}$$

$U_C = W_C$ (Distance pulley C lowered)

$$= 140\text{ N}\left[\frac{1}{2}(1.2 - 0.63958)\right]\text{m} = 39.229\text{ N}\cdot\text{m}$$

$$U_A = -68.67\text{ N }(\sin 15°)(0.6\text{ m}) = -10.6639\text{ N}\cdot\text{m}$$

$$U = T_2 - T_1 = U_C - U_A$$

$$\frac{1}{2}m_A v^2 - 0 = U_C - U_A$$

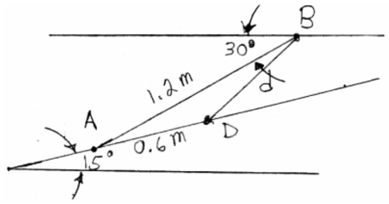
$$\frac{1}{2}(7\text{ kg})v^2 = (39.229 - 10.6639)\text{ N}\cdot\text{m}$$

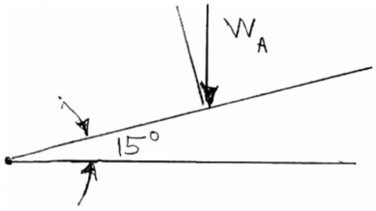
$$v^2 = 8.1615$$

$$v = 2.857\text{ m/s}$$

$v = 2.86\text{ m/s}$ ⦣ 15° ◀

Chapter 13, Solution 12.





$W_A = 7(9.81) = 68.67$ N

Given: Block A is released at the position shown at a velocity of 1.5 m/s up.

After moving 0.6 m the velocity is 3 m/s.

Find: work done by friction force on the block, $V_f J$

From the Law of Cosines

$$d^2 = (1.2)^2 + (0.6)^2 - 2(1.2)(0.6)(\cos 15°)$$

$$d^2 = 0.4091\,\text{m}^2$$

$$d = 0.63958\,\text{m}$$

$$U_C = 140\,\text{N}\left[\frac{1}{2}(1.2 - 0.63958)\right]\text{m} = 39.229\ \text{N}\cdot\text{m}$$

$$U_A = -68.67\ \text{N}(\sin 15°)(0.6\ \text{m}) = -10.664\ \text{N}\cdot\text{m}$$

$$U_C + U_A - U_{friction} = T_2 - T_1 = \frac{1}{2}\,m_A[v_2^2 - v_1^2]$$

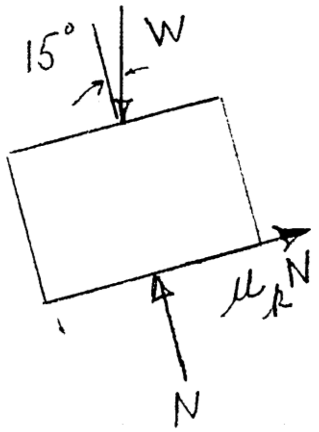
$$39.229 - 10.664 - U_{friction} = \frac{1}{2}(7\ \text{kg})[(3)^2 - (1.5)^2]\ \text{m}^2$$

$$-U_{friction} = -39.229 + 10.664 + 23.625$$

$$= -4.94\ \text{J}$$

$U_{friction} = -4.94$ J ◀

Chapter 13, Solution 13.



Given: At A, $v = v_0$

For AB, $\mu_k = 0.40$

At B, $v = 2$ m/s

Find: v_0

$$T_A = \frac{1}{2}mv_0^2 \qquad T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2\text{ m/s})^2$$

$$T_B = 2\text{ m}$$

$$U_{A-B} = (W\sin 15° - \mu_k\text{ N})(6\text{ m})$$

$$\nwarrow \Sigma F = 0 \qquad N - W\cos 15° = 0$$

$$N = W\cos 15°$$

$$U_{A-B} = W(\sin 15° - 0.40\cos 15°)(6\text{ m})$$

$$U_{A-B} = -(0.76531)W = -0.76531\,mg$$

$$T_A + U_{A-B} = T_B$$

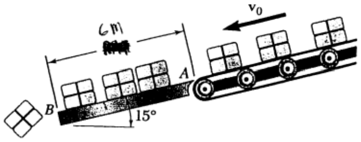
$$\frac{1}{2}mv_0^2 - 0.76531\,mg = 2\text{ m}$$

$$v_0^2 = (2)\left(2 + (0.76531)\left(9.81\text{ m/s}^2\right)\right)$$

$$v_0^2 = 19.0154$$

$v_0 = 4.36$ m/s ◀

Chapter 13, Solution 14.



Given: At A, $v = v_0$

At B, $v = 0$

For AB, $\mu_k = 0.40$

Find: v_0

$$T_A = \frac{1}{2}mv_0^2 \qquad T_B = 0$$

$$U_{A-B} = \left(W\sin 15° - \mu_k N\right)(6\text{ m})$$

$$\nwarrow \Sigma F = 0 \qquad N - W\cos 15° = 0$$

$$N = W\cos 15°$$

$$U_{A-B} = W\left(\sin 15° - 0.40\cos 15°\right)(6\text{ m})$$

$$U_{A-B} = -(0.76531)W = -0.76531\,mg$$

$$T_A + U_{A-B} = T_B$$

$$\frac{1}{2}mv_0^2 - 0.76531\,mg = 0$$

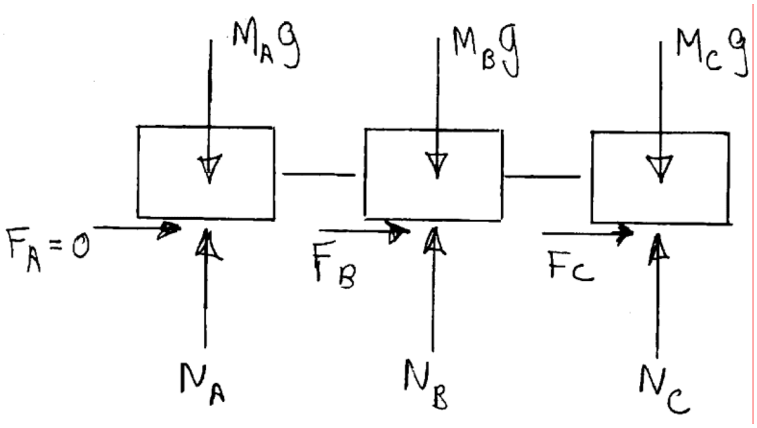
$$v_0^2 = (2)(0.76531)\left(9.81\text{ m/s}^2\right)$$

$$v_0^2 = 15.015$$

$v_0 = 3.87$ m/s ◀
Down to the left.

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 15.



Car C $+\uparrow\ \Sigma F_y - 0 \ \Rightarrow\ N_C - M_C g = 0 \ \Rightarrow\ N_C = (35\times 10^3\text{ kg})\,(9.81\text{ m/s}^2) = 343.35\times 10^3\text{ N}$

So, $F_C = (0.35)(343.35\times 10^3) = 120.173\times 10^3\text{ N}$

Car B $+\uparrow\ \Sigma F_y = 0 \ \Rightarrow\ N_B - M_B g = 0 \ \Rightarrow\ N_B = (45\times 10^3\text{ kg})(9.81\text{m/s}^2) = 441.45\times 10^3\text{ N}$

So, $F_B = 0.35(441.45\times 10^3) = 154.508\times 10^3\text{ N}$

Also,

$$v_1 = (54\text{ km/h})(1\text{h}/3600\text{s})(1000\text{ m/km}) = 15\text{ m/s}$$

(*a*) Work and energy for the train

$$T_1 + U_{1\to 2} = T_2$$

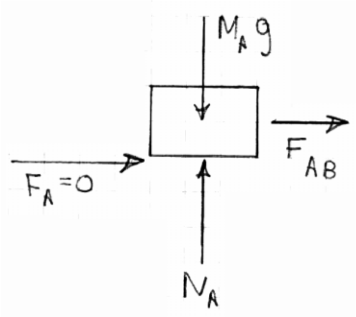
$$\frac{1}{2}(35\times 10^3 + 45\times 10^3 + 35\times 10^3)(15)^2 - (120.173\times 10^3 + 154.508\times 10^3)\ x = 0$$

$x = 47.10$ m

$x = 47.1$ m ◀

(*b*) Force in each coupling

Car A



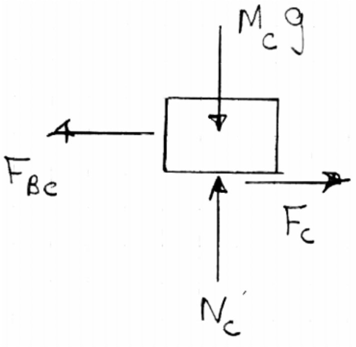
$T_1 + U_{1\to 2} = T_2$

$$\frac{1}{2}\left(35\times10^3\right)\left(15\right)^2 - F_{AB}\left(47.10\right) = 0$$

$F_{AB} = 83.599\times10^3$ N

$F_{AB} = 83.6$ kN ◀
Tension

Car C



$T_1 + U_{1\to x} = T_2$

$$\frac{1}{2}\left(35\times10^3\right)\left(15\right)^2 + \left(F_{BC} - 120.173\times10^3\right)\left(47.10\right) = 0$$

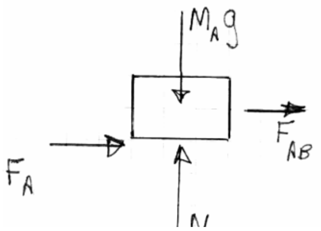
Solve for F_{BC}

$F_{BC} = 36.6\times10^3$ N

$F_{BC} = 36.6$ kN ◀
Tension

Chapter 13, Solution 16.

Car A



$$+\uparrow\ \Sigma F_y = 0 \Rightarrow N_A - M_A\, g = 0$$

$$N_A = \left(35\times10^3\right)(9.81) = 343.35\times10^3\ \text{N}$$

so,

$$F_A = (0.35)\left(343.35\times10^3\right) = 120.173\times10^3\ \text{N}$$

$$v_1 = \left(54\ \text{km/h}\right) = 15\ \text{m/s}$$

(*a*) Work - energy for the entire train

$$T_1 + U_{1\to2} = T_2$$

$$\frac{1}{2}\left[\left(35\times10^3\right)+\left(45\times10^3\right)+\left(35\times10^3\right)\right](15)^2 - 120.173\times10^3\, x = 0$$

$$x = 107.66\ \text{m}$$

$x = 107.7$ m ◀

(*b*) Force in each coupling

Car A

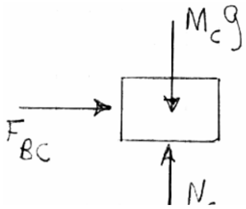
$$T_1 + U_{1\to2} = T_2$$

$$\frac{1}{2}\left(35\times10^3\right)(15)^2 - \left(F_{AB} + 120.173\times10^3\right)(107.66) = 0$$

$$F_{AB} = -83.60\times10^3\ \text{N}$$

$F_{AB} = 83.6$ kN ◀
Compression

Car C



$$T_1 + U_{1-2} = T_2$$

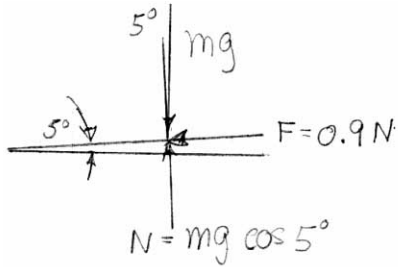
$$\frac{1}{2}\left(35\times10^3\right)(15)^2 + F_{BC}(107.66) = 0$$

$$F_{BC} = -36.57\times10^3\ \text{N}$$

$F_{BC} = 36.6$ kN ◀
Compression

Chapter 13, Solution 17.

Car B:



Given: Car *B* towing car *A* uphill at *A* constant speed of 30 ft/s

Car *B* skids to a stop. $\mu_k = 0.9$

Car *A* strikes rear of car *B*.

Find: Speed of car *A* before collision, v_A

Let d = Distance traveled by car *B* after braking.

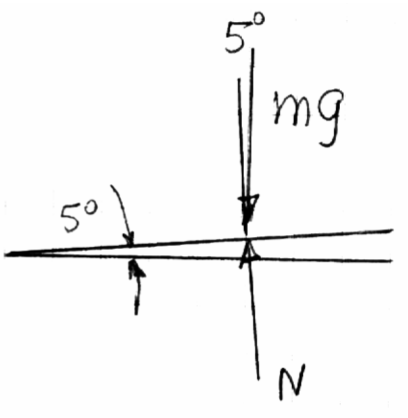
$$U_{1-2} = T_2 - T_1 \qquad -\frac{1}{2}\, m\, v_B^2 = \left(-mg\sin 5° - F\right) d$$

$$d = -\frac{\frac{1}{2}\,\cancel{m}\,(30)^2}{\cancel{m}g\sin 5° + 0.9\,\cancel{m}g\cos 5°}$$

$$d = \frac{450}{32.2\left(\sin 5° + 0.9\cos 5°\right)} = \frac{450}{(32.2)(0.9837)}$$

$$d = 14.206\ \text{ft traveled by } B$$

Car A:



For car *A*, travel to contact

$$U_{1\to C} = T_A - T_1 = \frac{1}{2}\, mv_A^2 - \frac{1}{2}\, mv_1^2$$

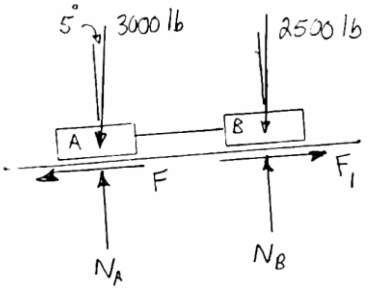
$$\left(-mg\sin 5°\right)\left(d + 15\right) = \frac{1}{2}\, mv_A^2 - \frac{1}{2}\, m\,(30)^2$$

$$\frac{1}{2}\, v_A^2 - 450 = \left(-32.2\sin 5°\right)\left(14.206 + 15\right)$$

$$\frac{1}{2}\, v_A^2 = 368.036$$

$$v_A = 27.13 \qquad\qquad v_A = 27.1\ \text{ft/s} \blacktriangleleft$$

Chapter 13, Solution 18.



$N_B = 2500\cos 5°$

$N_A = 3000\cos 5°$

$F = 0.8\,N_A$

Given: Car *B* tows car *A* at 30 ft/s uphill.

Car *A* brakes for 4 wheels skid $\mu_k = 0.8$

Car *B* continues in same gear and throttle setting.

Find: (*a*) distance, *d*, traveled to stop

(*b*) tension in cable

(*a*) F_1 = Traction force (from equilibrium)

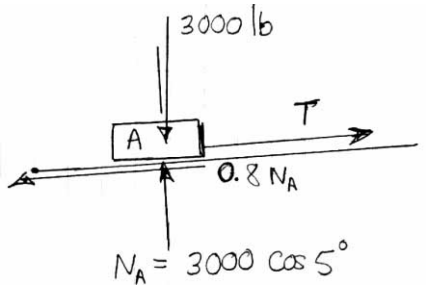
$$F_1 = (3000)\sin 5° + (2500)\sin 5°$$

$$= 5500\sin 5°$$

For system *A* + *B*

$$U_{1\to 2} = \left[\left(F_1 - 3000\sin 5° - 2500\sin 5°\right) - F\right]d$$

$$T_2 - T_1 = 0 - \frac{1}{2}m_{A+B}\,v^2 = -\frac{1}{2}\left(\frac{5500}{32.2}\right)(30)^2$$



Since $\left(F_1 - 3000\sin 5° - 2500\sin 5°\right) = 0$

$$-Fd = -0.8\left(3000\cos 5°\right)d = -76863\text{ ft}\cdot\text{lb}$$

$d = 32.1$ ft ◀

(*b*) cable tension, *T*

$$U_{1\to 2} = \left(T - 0.8N_A - W_A\sin 5°\right)(32.149) = T_2 - T_1$$

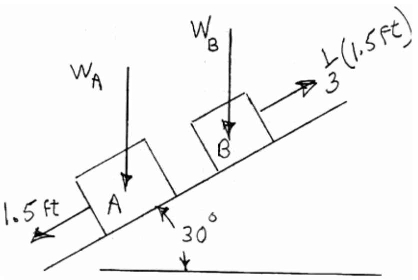
$$\left(T - 0.8(3000)(\cos 5°) - 3000\sin 5°\right)(32.149) = \frac{3000(30)^2}{2(32.2)}$$

$$(T - 2652.3) = -1304$$

$$= 1348\text{ lb}$$

$T = 1348$ lb ◀

Chapter 13, Solution 19.



Given: Blocks A, B released from rest and friction and masses of pulleys neglected.

Find: (a) Velocity of block A, v_A, after moving down $d_A = 1.5$ ft.

(b) The tension in the cable.

(a) constraint $\quad v_A + 3\,v_B = 0 \qquad v_B = \frac{1}{3}\,v_A$

Also, $\quad d_B = \frac{1}{3}\,d_A$

$$U_{1\to 2} = W_A\left(\sin 30°\right) d_A - W_B\left(\sin 30°\right) d_B$$

$$= 20\left(\sin 30°\right)\left(1.5\right) - 16\left(\sin 30°\right)\left(\frac{1.5}{3}\right)$$

$$= 11\text{ ft}\cdot\text{lb}$$

$$T_1 = 0,\ T_2 = \frac{1}{2}\,m_A\,v_A^2 + \frac{1}{2}\,m_B\,v_B^2$$

$$= \frac{1}{2}\left(\frac{20}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{16}{32.2}\right)\left(\frac{v_A}{3}\right)^2 = 0.33816\,v_A^2$$

$$U_{1\to 2} = T_2 - T_1\ ;\ 11 = 0.33816\,v_A^2$$

$$v_A = 5.703 \qquad v_A = 5.70\text{ ft/s} \swarrow 30° \blacktriangleleft$$

(b) For A alone

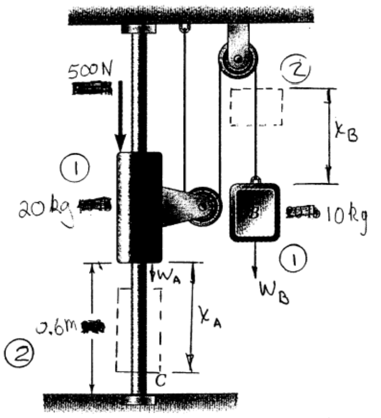
$$U_{1\to 2} = W_A\left(\sin 30°\right) d_A - T\left(d_A\right) = \frac{1}{2}\,m_A\left(v_A\right)^2$$

$$20\left(0.5\right)\left(1.5\right) - T\,\left(1.5\right) = \frac{1}{2}\left(\frac{20}{32.2}\right)\left(5.703\right)^2 = 10.102$$

$$T = 3.265\text{ ft}\cdot\text{lb}$$

$$T = 3.27\text{ ft}\cdot\text{lb} \blacktriangleleft$$

Chapter 13, Solution 21.



Given: System at rest when 500 N force is applied to collar *A*. No friction. Ignore pulleys mass.

Find: (*a*) Velocity, v_A of *A* just before it hits *C*.

(*b*) v_A If counter weight *B* is replaced by a 98.1 N downward force.

Kinematics

$$X_B = 2X_A$$

$$v_B = 2v_A$$

(*a*) Blocks *A* and *B*

$$T_1 = 0 \qquad T_2 = \frac{1}{2}m_B v_B^2 + \frac{1}{2}m_A v_A^2$$

$$T_2 = \frac{1}{2}(10\text{ kg})(2v_A)^2 + \frac{1}{2}(20\text{ kg})\left(v_A{}^2\right)$$

$$T_2 = (30\text{ kg})(v_A)^2$$

$$U_{1-2} = (500)X_A + (W_A)(X_A) - (W_B)(X_B)$$

$$U_{1-2} = (500\text{ N})(0.6\text{ m}) + \left(20\text{ kg}\times 9.81\text{ m/s}^2\right)(0.6\text{ m})$$

$$-\left(10\text{ kg}\times 9.81\text{ m/s}^2\right)(1.2\text{ m})$$

$$U_{1-2} = 300 + 117.72 - 117.72 = 300\text{ J}$$

$$T_1 + U_{1-2} = T_2 \qquad 0 + 300\text{ J} = (30\text{ kg})v_A^2$$

$$v_A^2 = 10$$

$$v_A = 3.16\text{ m/s} \blacktriangleleft$$

(*b*) Since the 10 kg mass at *B* is replaced by a 98.1 N force, kinetic energy at ② is,

$$T_2 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(20\text{ kg})v_A^2 \qquad T_1 = 0$$

The work done is the same as in part (*a*)

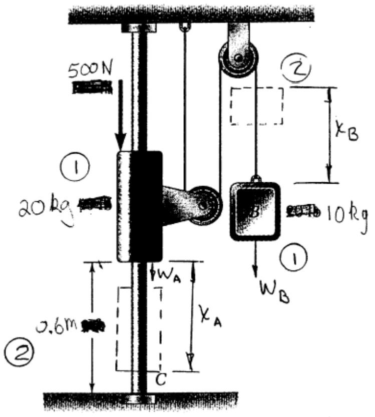
$$U_{1-2} = 300\text{ J}$$

$$T_1 + U_{1-2} = T_2 \qquad 0 + 300\text{ J} = (10\text{ kg})v_A^2$$

$$v_A^2 = 30$$

$$v_A = 5.48\text{ m/s} \blacktriangleleft$$

Chapter 13, Solution 21.



Given: System at rest when 500 N force is applied to collar *A*. No friction. Ignore pulleys mass.

Find: (*a*) Velocity, v_A of *A* just before it hits *C*.

(*b*) v_A If counter weight *B* is replaced by a 98.1 N downward force.

Kinematics

$$X_B = 2X_A$$

$$v_B = 2v_A$$

(*a*) Blocks *A* and *B*

$$T_1 = 0 \qquad T_2 = \frac{1}{2}m_B v_B^2 + \frac{1}{2}m_A v_A^2$$

$$T_2 = \frac{1}{2}(10\text{ kg})(2v_A)^2 + \frac{1}{2}(20\text{ kg})\left(v_A{}^2\right)$$

$$T_2 = (30\text{ kg})(v_A)^2$$

$$U_{1-2} = (500)X_A + (W_A)(X_A) - (W_B)(X_B)$$

$$U_{1-2} = (500\text{ N})(0.6\text{ m}) + \left(20\text{ kg} \times 9.81\text{ m/s}^2\right)(0.6\text{ m})$$

$$-\left(10\text{ kg} \times 9.81\text{ m/s}^2\right)(1.2\text{ m})$$

$$U_{1-2} = 300 + 117.72 - 117.72 = 300\text{ J}$$

PROBLEM 13.21 CONTINUED

$$T_1 + U_{1-2} = T_2 \qquad 0 + 300\text{ J} = (30\text{ kg})v_A^2$$

$$v_A^2 = 10$$

$$v_A = 3.16\text{ m/s} \blacktriangleleft$$

(*b*) Since the 10 kg mass at *B* is replaced by a 98.1 N force, kinetic energy at ② is,

$$T_2 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(20\text{ kg})v_A^2 \qquad T_1 = 0$$

The work done is the same as in part (*a*)

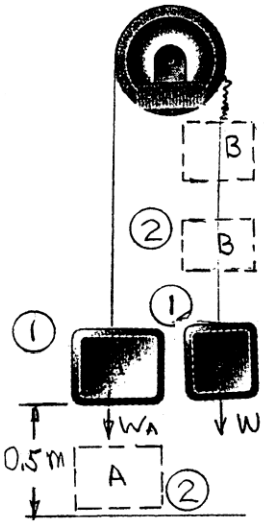
$$U_{1-2} = 300\text{ J}$$

$$T_1 + U_{1-2} = T_2 \qquad 0 + 300\text{ J} = (10\text{ kg})v_A^2$$

$$v_A^2 = 30$$

$$v_A = 5.48\text{ m/s} \blacktriangleleft$$

Chapter 13, Solution 22.



Given: $m_A = 10 \text{ kg};\ m_B = 4 \text{ kg};\ h = 0.5 \text{ m}$

System released from rest.

Block A hits the ground without rebound.

Block B reaches a height of 1.18 m.

Find: (*a*) v_A just before block A hits the ground.

(*b*) Energy, E_P, dissipated by the pulley friction.

(*a*) v_B at ② $= v_A$ at ② just before impact.

from ② to ③; Block B

$$T_3 = 0 \qquad T_2 = \frac{1}{2}m_B v_B^2 = \frac{1}{2}(4)v_B^2 = 2v_B^2$$

Tension in the cord is zero, thus

$$U_{2-3} = -(4 \text{ kg})(9.81 \text{ m/s}^2)(0.18 \text{ m}) = -7.0632 \text{ J}$$

$$T_2 + U_{2-3} = T_3; \qquad 2v_B^2 = 7.0632; \qquad v_B^2 = 3.5316$$

$$v_B^2 = v_A^2 = 3.5316 \qquad v_B = v_A = 1.8793 \qquad v_A = 1.879 \text{ m/s} \blacktriangleleft$$

(*b*) From ① to ② Blocks A and B,

$$T_1 = 0 \qquad T_2 = \frac{1}{2}(m_A + m_B)v_2^2$$

Just before impact $v_2 = v_B = v_A = 1.793 \text{ m/s}$

$$T_2 = \frac{1}{2}(10+4)(1.8793)^2 = 24.722 \text{ J}$$

$$U_{1-2} = W_A(0.5) - W_B(0.5) - E_P;$$

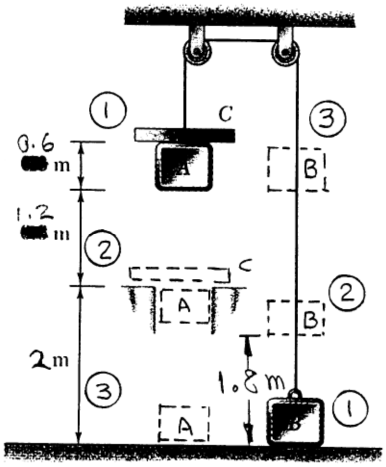
$$E_P = \text{Energy dissipated by pulley}$$

$$U_{1-2} = \left[(9.81 \text{ m/s}^2)(10-4)\text{kg}\right](0.5 \text{ m}) - E_P = 29.43 - E_P$$

$$T_1 + U_{1-2} = T_2; \qquad 0 + 29.43 - E_P = 24.722$$

$$E_P = 4.708 \qquad E_P = 4.71 \text{ J} \blacktriangleleft$$

Chapter 13, Solution 23.



Given: $m_A = 8 \text{ kg}; \quad m_B = 10 \text{ kg}; \quad m_C = 6 \text{ kg}$

System released from rest.

Collar C removed after blocks move 1.8 m.

Find: v_A, just before it strikes the ground.

Position 1 to position 2

$$v_1 = 0 \qquad T_1 = 0$$

At 2, before C is removed from the system

$$T_2 = \frac{1}{2}\left(m_A + m_B + m_C\right)v_2^2$$

$$T_2 = \frac{1}{2}\left(24 \text{ kg}\right)v_2^2 = 12v_2^2$$

$$U_{1-2} = \left(m_A + m_C - m_B\right)g\left(1.8 \text{ m}\right)$$

$$U_{1-2} = \left(8 + 6 - 10\right)g\left(1.8 \text{ m}\right) = 70.632 \text{ J}$$

$$T_1 + U_{1-2} = T_2; \qquad 0 + 70.632 = 12v_2^2$$

$$v_2^2 = 5.886$$

Position 2 to position 3

$$T_2' = \frac{1}{2}\left(m_A + m_B\right)v_2^2 = \frac{18}{2}\left(5.886\right) = 52.974$$

$$T_3 = \frac{1}{2}\left(m_A + m_B\right)v_3^2 = 9v_3^2$$

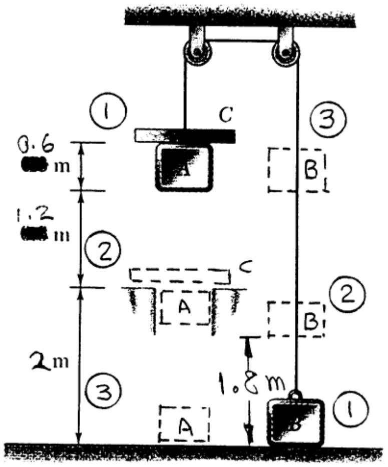
$$U_{2'-3} = \left(m_A - m_B\right)g\left(2 - 0.6\right) = \left(-2 \text{ kg}\right)\left(9.81 \text{ m/s}^2\right)\left(1.4 \text{ m}\right)$$

$$U_{2'-3} = -27.468 \text{ J}$$

$$T_2' + U_{2'-3} = T_3 = 52.974 - 27.468 = 9v_3^2$$

$$v_3^2 = 2.834 \qquad v_3 = 1.68345 \qquad v_A = 1.683 \text{ m/s} \blacktriangleleft$$

Chapter 13, Solution 23.



Given: $m_A = 8\text{ kg};\quad m_B = 10\text{ kg};\quad m_C = 6\text{ kg}$

System released from rest.

Collar C removed after blocks move 1.8 m.

Find: v_A, just before it strikes the ground.

Position 1 to position 2

$$v_1 = 0 \qquad T_1 = 0$$

At 2, before C is removed from the system

$$T_2 = \frac{1}{2}\left(m_A + m_B + m_C\right)v_2^2$$

$$T_2 = \frac{1}{2}\left(24\text{ kg}\right)v_2^2 = 12v_2^2$$

$$U_{1-2} = \left(m_A + m_C - m_B\right)g\left(1.8\text{ m}\right)$$

$$U_{1-2} = \left(8 + 6 - 10\right)\text{g}\left(1.8\text{ m}\right) = 70.632\text{ J}$$

$$T_1 + U_{1-2} = T_2; \qquad 0 + 70.632 = 12v_2^2$$

$$v_2^2 = 5.886$$

Position 2 to position 3

$$T_2' = \frac{1}{2}\left(m_A + m_B\right)v_2^2 = \frac{18}{2}\left(5.886\right) = 52.974$$

$$T_3 = \frac{1}{2}\left(m_A + m_B\right)v_3^2 = 9v_3^2$$

$$U_{2'-3} = \left(m_A - m_B\right)g\left(2 - 0.6\right) = \left(-2\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(1.4\text{ m}\right)$$

$$U_{2'-3} = -27.468\text{ J}$$

$$T_2' + U_{2'-3} = T_3 = 52.974 - 27.468 = 9v_3^2$$

$$v_3^2 = 2.834 \qquad v_3 = 1.68345 \qquad v_A = 1.683\text{ m/s} \blacktriangleleft$$

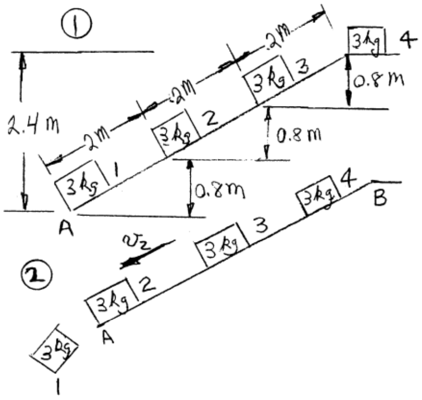
Chapter 13, Solution 24.

Given: Conveyor is disengaged, packages held by friction and system is released from rest. Neglect mass of belt and rollers. Package 1 leaves the belt as package 4 comes onto the belt.

Find: (*a*) Velocity of package 2 as it leaves the belt at *A*.

(*b*) Velocity of package 3 as it leaves the belt at *A*.

(*a*) Package 1 falls off the belt, and 2, 3, 4 move down.



$$\frac{2.4}{3} = 0.8 \text{ m}$$

$$T_2 = 3\left[\frac{1}{2}mv_2^2\right]$$

$$T_2 = \frac{3}{2}(3 \text{ kg})v_2^2$$

$$T_2 = 4.5v_2^2$$

$$U_{1-2} = (3)(W)(0.8) = (3)(3 \text{ kg}) \times 9.81 \text{ m/s}^2 (0.8)$$

$$U_{1-2} = 70.632 \text{ J}$$

$$T_1 + U_{1-2} = T_2 \qquad 0 + 70.632 = 4.5v_2^2$$

$$v_2^2 = 15.696$$

$$v_2 = 3.9618 \qquad\qquad v_2 = 3.96 \text{ m/s} ◀$$

Chapter 13, Solution 25.

Work and energy $T_1 + U_{1\to 2} = T_2$ (1)

Where $T_1 = 0;\ \ T_2 = 0$

Work

Outer spring $V_{1-2} = -\dfrac{1}{2}k_1 x^2 = -\dfrac{1}{2}\left(3000\dfrac{\text{N}}{\text{m}}\right)\left(1.5\text{ m}\right)^2$

$= -33.75\text{ J}$

Inner spring $U_{1-2} = -\dfrac{1}{2}k_2 x^2 = -\dfrac{1}{2}\left(10{,}000\dfrac{\text{N}}{\text{m}}\right)\left(0.06\text{ m}\right)^2$

$= -18\text{ J}$

Gravity $U_{1-2} = mg\left(h + 0.15\right)$

$= \left(8\right)\left(9.81\right)\left(h + 0.15\right) = 78.48\,h + 11.722$

Total work $U_{1-2} = -33.75 - 18 + 78.48\,h + 11.772$

$= -39.978 + 78.48\,h$

Substituting into (1)

$0 - 39.978 + 78.48\,h = 0$

$h = 0.5094\text{ m}$

$h = 509\text{ mm}$ ◀

Chapter 13, Solution 26.

Work and energy

Assume $x > 0.09$ m

$$T_1 + U_{1\to 2} = T_2 \qquad (1)$$

Where $T_1 = 0;\ \ T_2 = 0$

Work

Outer spring $U_{1\to 2_1} = -\frac{1}{2}k_1 x^2 = \frac{1}{2}(3000)x^2 = -1500x^2$

Inner spring $U_{1\to 2_2} = -\frac{1}{2}k_2(x - 0.09)^2 = -5000(x - 0.09)^2$

Gravity $U_{1\to 2} = mg(x + 0.6)$

$$= (8)(9.81)(x + 0.6) = 78.48x + 47.09$$

Total work

$$U_{1\to 2} = -1500x^2 - 5000(x - 0.09)^2 + 78.48x + 47.09$$

Substitute into (1)

$$-6500x^2 + 978.48x + 6.588 = 0$$

Solve

$x = 0.1570$ m or -0.00646

Reject negative solution $x = 157.0$ mm ◀

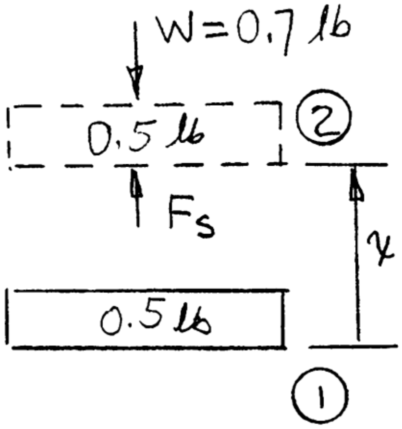
Chapter 13, Solution 27.

Given: A 0.7 lb block rests on a 0.5 lb block which is not attached to a spring of constant 9 lb/ft; upper block is suddenly removed.

Find: (*a*) v_{max} of 0.5 lb block

(*b*) maximum height reached by the 0.5 lb block

(*a*)



At the initial position (1), the force in the spring equals the weight of both blocks, i.e., 1.2 lb.

Thus at a distance *x*, the force in the spring is,

$$F_s = 1.2 - kx$$

$$F_s = 1.2 - 9x$$

Max velocity of the 0.5 lb block occurs while the spring is still in contact with the block.

$$T_1 = 0 \qquad T_2 = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{0.5}{g}\right)v^2 = \frac{0.25}{g}v^2$$

$$U_{1-2} = \int_0^x (1.2 - 9x)\,dx - 0.5x = 0.7x - \frac{9}{2}x^2$$

$$T_1 + U_{1-2} = T_2 = 0.7x - \frac{9}{2}x^2 = \frac{0.25}{g}v^2$$

$$v^2 = 4g\left[0.7x - \frac{9}{2}x^2\right]$$

$$V_{\text{max}} \text{ when } \frac{dv}{dx} = 0 = 0.7 - 9x \Rightarrow x = 0.077778 \text{ ft}$$

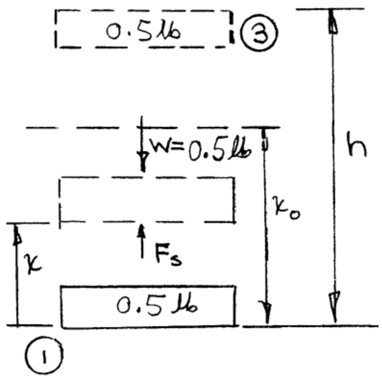
$$v_{\text{max}}^2 = 4g\left[0.7(0.077778) - \frac{9}{2}(0.077778)^2\right]$$

$$v_{\text{max}}^2 = 3.5063$$

$$v_{\text{max}} = 1.87249$$

$v_{\text{max}} = 1.872$ ft/s ↑ ◀

(*b*)



$x_0 =$ Initial compression

$$x_0 = \frac{1.2 \text{ lb}}{9 \text{ lb/ft}} = 0.133333 \text{ ft}$$

$$F_s = 1.2 - 9x$$

$$T_1 = 0, \; T_3 = 0$$

$$U_{1-3} = \int_0^{x_0} F_s dx - 0.5h$$

$$U_{1-3} = \int_0^{x_0} (1.2 - 9x)\, dx - 0.5h$$

$$= 1.2x_0 - \frac{9}{2}x_0^2 - 0.5h$$

$$= 1.2(0.133333) - \frac{9}{2}(0.133333)^2 - 0.5h$$

$$= 0.08 - 0.5h$$

$$T_1 + U_{1-3} = T_3: \quad 0 + (0.08 - 0.5h) = 0$$

$$h = 0.16 \text{ ft}$$

$h = 1.920$ in. ◀

Chapter 13, Solution 28.

(*a*) Same as 13.25 solution for Part (*a*) $v_{\max} = 1.872 \text{ ft/s} \uparrow$ ◀

(*b*) With 0.5 lb block attached to the spring, refer to figure in (*b*) of Problem 13.27.

$$T_1 = 0 \qquad T_3 = 0 \qquad U_{1-3} = \int_0^h (1.2 - 9x)\,dx - 0.5h$$

Since the spring remains attached to the 0.5 lb block,
the integration must be carried out for the total distance, *h*.

$$T_1 + U_{1-3} = T_3 \qquad 0 + 0.7h - \frac{9}{2}h^2 = 0$$

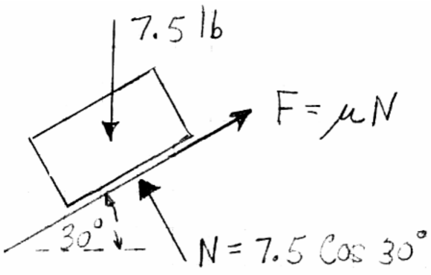
$$h = \left(\frac{2}{9 \text{ lb/ft}}\right)(0.7 \text{ lb}) = 0.155556 \text{ ft}$$

$$h = 1.86667 \text{ in.}$$

$h = 1.867$ in. ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 29.



Position 1, initial condition

Position 2, spring deflected 5 inches

Position 3, initial contact of spring with collar

$$U_{1\to 2} = \underset{\text{(Friction)}}{-F\left(\frac{18+5}{12}\right)} - \underset{\text{(Spring)}}{\frac{1}{2}(60)\left(\frac{5}{12}\right)^2} + \underset{\text{(Gravity)}}{7.5\left(\frac{18+5}{12}\right)\sin 30°}$$

$$T_1 = T_2 = 0, \therefore U_{1-2} = 0$$

$$0 = -\mu(7.5)(0.866)\left(\frac{23}{12}\right) - \frac{1}{2}(60)\left(\frac{5}{12}\right)^2 + 7.5\left(\frac{23}{12}\right)(0.5)$$

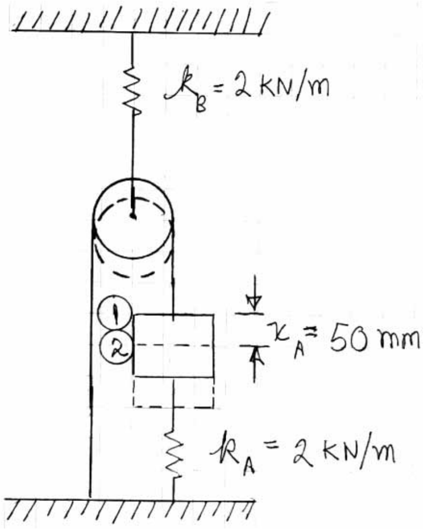
(*a*) $\mu = 0.1590$ ◀

(*b*) Max speed occurs just before contact with the spring

$$U_{1\to 3} = -\mu(7.5)(0.866)\left(\frac{18}{12}\right) + 7.5\left(\frac{18}{12}\right)(0.5) = T_3 = \frac{1}{2}\left(\frac{7.5}{32.2}\right)v_{\max}^2$$

$v_{\max} = 5.92$ ft/s ◀

Chapter 13, Solution 30.



(*a*) $W =$ Weight of the block $= 10\,(9.81) = 98.1$ N

$$x_B = \frac{1}{2}x_A$$

$$U_{1-2} = W(x_A) - \frac{1}{2}k_A(x_A)^2 - \frac{1}{2}k_B(x_B)^2$$

(Gravity) (Spring *A*) (Spring *B*)

$$U_{1-2} = (98.1\text{ N})(0.05\text{ m}) - \frac{1}{2}(2000\text{ N/m})(0.05\text{ m})^2$$

$$-\frac{1}{2}(2000\text{ N/m})(0.025\text{ m})^2$$

$$U_{1-2} = \frac{1}{2}(m)\,v^2 = \frac{1}{2}(10\text{ kg})\,v^2$$

$$4.905 - 2.5 - 0.625 = \frac{1}{2}(10)\,v^2$$

$v = 0.597$ m/s ◀

(*b*) Let $x =$ Distance moved down by the 10 kg block

$$U_{1-2} = W(x) - \frac{1}{2}k_A(x)^2 - \frac{1}{2}k_B\left(\frac{x}{2}\right)^2 = \frac{1}{2}(m)\,v^2$$

$$\frac{d}{dx}\left[\frac{1}{2}(m)\,v^2\right] = 0 = W - k_A(x) - \frac{k_B}{8}(2x)$$

$$0 = 98.1 - 2000\left(x\right) - \frac{2000}{8}\left(2x\right) = 98.1 - \left(2000 + 250\right)x$$

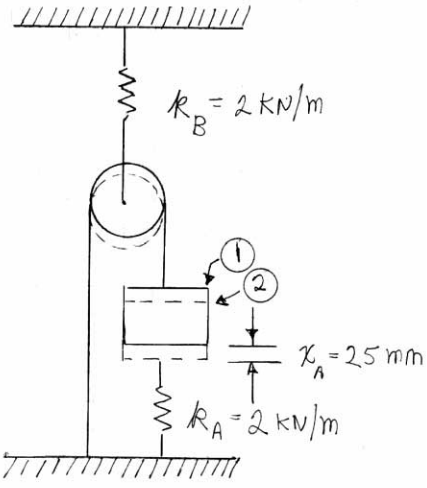
$$x = 0.0436 \text{ m } \left(43.6 \text{ mm}\right)$$

For $x = 0.0436$, $U = 4.2772 - 1.9010 - 0.4752 = \frac{1}{2}\left(10\right)v^2$

$$v_{\text{max}} = 0.6166 \text{ m/s}$$

$v_{\text{max}} = 0.617 \text{ m/s}$ ◀

Chapter 13, Solution 31.



(*a*) W = Weight of the block = (10)(9.81) = 98.1 N

$$x_B = \frac{1}{2}x_A$$

Block moves down at release after spring *A* is stretched 25 mm

$$U_{1-2} = W(x_A) + \frac{1}{2}k_A(x_A)^2 - \frac{1}{2}k_B(x_B)^2$$

(Gravity) (Spring *A*) (Spring *B*)

$$U_{1-2} = (98.1 \text{ N})(0.025 \text{ m}) + \frac{1}{2}(2000 \text{ N/m})(0.025 \text{ m})^2$$

$$-\frac{1}{2}(2000 \text{ N/m})(0.0125 \text{ m})^2$$

$$= \frac{1}{2}(m)v^2 = \frac{1}{2}(10 \text{ kg})v^2$$

$$U_{1-2} = 2.4525 + 0.625 - 0.15625 = \frac{1}{2}(10)v^2$$

$v = 0.764$ m/s

$v = 0.764$ m/s ◀

(*b*) Let x = Distance moved down by the 10 kg block (for $x > 25$ mm)

$$U_{1-2} = Wx + \frac{1}{2}k_A(0.025) - \frac{1}{2}k_A(x - 0.025)^2$$

$$-\frac{1}{2}k_B\left(\frac{x}{2}\right)^2 = \frac{1}{2}(M)v^2$$

$$U_{1-2} = 98.1x + \frac{1}{2}(2000)(0.025)^2 - \frac{1}{2}(2000)(x - 0.025)^2 - \frac{1}{2}(2000)\left(\frac{x}{2}\right)^2$$

$$= \frac{1}{2}(10)v^2$$

$$0 = \frac{d}{dx}\left[\frac{1}{2}(10)v^2\right] = 98.1 - 2000(x - 0.025) - 2000\left(\frac{x}{4}\right)$$

$$= 98.1 - 2000x + 50 - 500x$$

$$x = \frac{148.1}{2500} = 0.05924 \text{ m } (= 59.24 \text{ mm})$$

For $x = 0.05924$ m

$$U_{1-2} = 98.1(0.05924) + 0.625 - 1000(0.03424)^2 - 0.87734$$

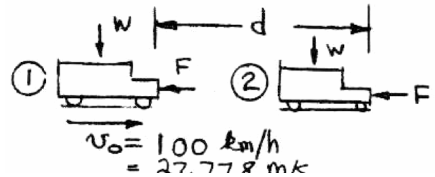
$$5.8114 + 0.625 - 1.1724 - 0.87734 = \frac{1}{2}(10)v^2$$

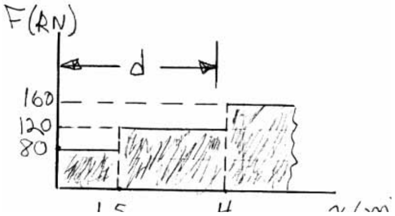
$$v_{\max} = 0.937 \text{ m/s}$$

$v_{\max} = 0.937$ m/s ◀

Chapter 13, Solution 32.

(*a*)





Assume auto stops in $1.5 < d < 4$ m.

$$v_1 = 27.778 \text{ m/s} \qquad T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1000 \text{ kg})(27.778 \text{ m/s})^2$$

$$= 385809 \text{ J} = 385.81 \text{ kJ}$$

$$T_2 = 0 \qquad v_2 = 0$$

$$U_{1-2} = (80 \text{ kN})(1.5 \text{ m}) + (120 \text{ kN})(d - 1.5) = 120 + 120d - 180 = 120d - 60$$

$$T_1 + U_{1-2} = T_2 \qquad 385.81 = 120d - 60 \qquad d = 3.715 \text{ m}$$

$d = 3.72$ m ◀

Assumption that $d < 4$ m is O.K.

(*b*) Maximum deceleration occurs when F is largest.

For $d = 3.3401$ m, $F = 120$ kN, thus $F = ma_D$

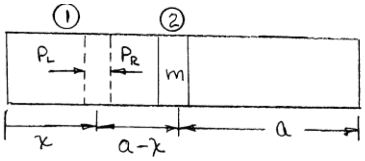
$$(120{,}000 \text{ N}) = (1000 \text{ kg})(a_D)$$

$$a_D = 120 \text{ m/s}^2$$

$a_D = 120.0 \text{ m/s}^2$ ◀

Chapter 13, Solution 33.

Pressures vary inversely as the volume



$$\frac{P_L}{P} = \frac{Aa}{Ax} \qquad P_L = \frac{Pa}{x}$$

$$\frac{P_R}{P} = \frac{Aa}{A(2a - x)} \qquad P_R = \frac{Pa}{(2a - x)}$$

Initially at ① $\quad v = 0 \qquad x = \frac{a}{2}$

$$T_1 = 0$$

At ②, $\quad x = a,\ T_2 = \frac{1}{2}mv^2$

$$u_{1-2} = \int_{\frac{a}{2}}^{a}(P_L - P_R)\,Adx = \int_{\frac{a}{2}}^{a} PaA\left[\frac{1}{x} - \frac{1}{2a - x}\right]dx$$

$$u_{1-2} = paA\left[\ln x + \ln(2a - x)\right]_{\frac{a}{2}}^{a}$$

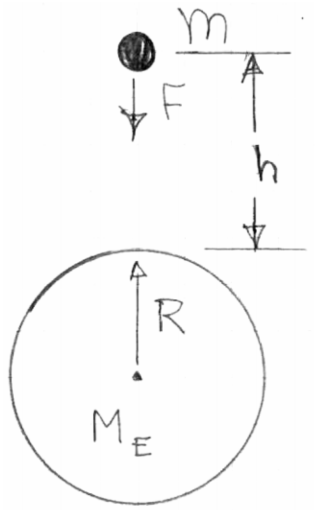
$$u_{1-2} = paA\left[\ln a + \ln a - \ln\left(\frac{a}{2}\right) - \ln\left(\frac{3a}{2}\right)\right]$$

$$u_{1-2} = paA\left[\ln a^2 - \ln\frac{3a^2}{4}\right] = paA\ln\left(\frac{4}{3}\right)$$

$$T_1 + U_{1-2} = T_2 \qquad 0 + paA\ln\left(\frac{4}{3}\right) = \frac{1}{2}mv^2$$

$$v^2 = \frac{2paA\ln\left(\frac{4}{3}\right)}{m} = 0.5754\frac{paA}{m} \qquad v = 0.759\sqrt{\frac{paA}{m}} \blacktriangleleft$$

Chapter 13, Solution 34.



$$F = \frac{GM_E m}{(h+R)^2} = \frac{GM_E m/R^2}{\left(\frac{h}{R}+1\right)^2} = mg_h$$

At earths' surface $(h = 0)$ $\qquad \dfrac{GM_E\, m}{R^2} = mg_0$

$$\frac{GM_E}{R^2} = g_0 \qquad g_h = \frac{\frac{GM_E}{R^2}}{\left(\frac{h}{R}+1\right)^2}$$

Thus $\qquad g_h = \dfrac{g_0}{\left(\frac{h}{R}+1\right)^2}$

$$R = 6370 \text{ km}$$

At altitude h, "true" weight $\qquad F = mg_h = W_T$

Assume weight $\qquad W_0 = mg_0$

$$\text{Error} = E = \frac{W_0 - W_T}{W_0} = \frac{mg_0 - mg_h}{mg_0} = \frac{g_0 - g_h}{g_0}$$

$$g_h = \frac{g_0}{\left(\frac{h}{R}+1\right)^2} \qquad E = \frac{g_0 - \frac{g_0}{\left(1+\frac{h}{R}\right)^2}}{g_0} = \left[1 - \frac{1}{\left(1+\frac{h}{R}\right)^2}\right]$$

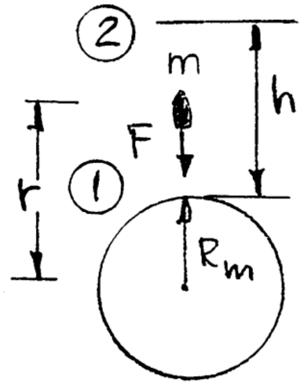
(a) $\quad h = 1$ km: $\quad P = 100E = 100\left[1 - \dfrac{1}{\left(1+\frac{1}{6370}\right)^2}\right]$

$P = 0.0314\%$ ◀

(b) $\quad h = 1000$ km: $\quad P = 100E = 100\left[1 - \dfrac{1}{\left(1+\frac{1000}{6370}\right)^2}\right]$

$P = 25.3\%$ ◀

Chapter 13, Solution 35.



Newtons law of gravitation

$$T_1 = \frac{1}{2}mv_0^2 \qquad T_2 = \frac{1}{2}mv^2$$

$$u_{1-2} = \int_{R_m}^{R_m+h_n}\left(-F_n\right)dr \qquad F_n = \frac{mg_mR^2{}_m}{r^2}$$

$$u_{1-2} = -mg_mR^2{}_m\int_{R_m}^{R_m+h_n}\frac{dr}{r^2}$$

$$u_{1-2} = mg_mR^2{}_m\left(\frac{1}{R_m}-\frac{1}{R_m+h_n}\right)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}mv_0^2 + mg_m\left(R_m - \frac{R_m}{R_m+h_n}\right) = \frac{1}{2}mv^2$$

$$h_n = \frac{\left(v_0^2-v^2\right)}{2g_m}\left[\frac{R_m}{R_m-\frac{\left(v_0^2-v^2\right)}{2gm}}\right] \qquad (1)$$

Uniform gravitational field

$$T_1 = \frac{1}{2}mv_0^2 \qquad T_2 = mv^2$$

$$u_{1-2} = \int_{R_m}^{R_m+h_n}\left(-F_u\right)dr = -mg_m\left(R_m+h_u-R_m\right) = -mgh_u$$

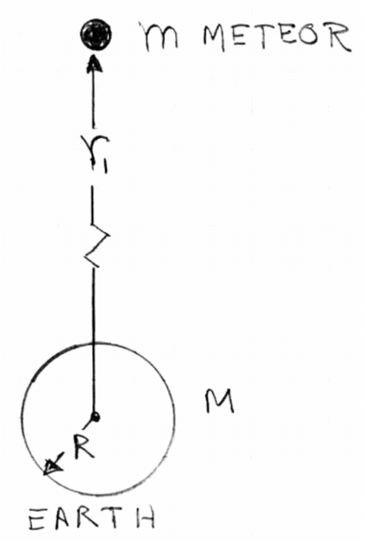
$$T_1 + u_{1-2} = T_2 \qquad \frac{1}{2}mv_0^2 - mg_mh_u = \frac{1}{2}mv^2$$

$$h_u = \frac{\left(v_0^2-v^2\right)}{2g_m} \qquad (2)$$

Divide (1) *by* (2)

$$\frac{h_n}{h_u} = \frac{1}{1-\frac{\left(v_0^2-v^2\right)}{\left(2g_mR_m\right)}} \blacktriangleleft$$

Chapter 13, Solution 36.



$$R = (3960\text{ mi})(5280\text{ ft/mi}) = 20.9088 \times 10^6\text{ ft}$$

$$U_{1-2} = T_2 - T_1 = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

Since r_1 is very large,

$$\frac{GMm}{r_1} \approx 0 \qquad \text{thus} \qquad T_1 \approx 0$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r_2}, \ \frac{v^2}{2} = g\frac{R^2}{r_2}$$

$$v^2 = \frac{2gR^2}{r_2} = \frac{2\left(32.2\text{ ft/s}^2\right)\left(20.9088 \times 10^6\text{ ft}\right)^2}{r_2}$$

(*a*) For $r_2 = 20.9088 \times 10^6\text{ ft} + ((620\text{ mi})(5280\text{ ft/mi}))$

$= 24.1824 \times 10^6\text{ ft}$

$v = 34{,}121\text{ ft/s}$ $\qquad v = 6.46\text{ mi/s}$ ◀

(*b*) For $r_2 = 20.9088 \times 10^6\text{ ft} + ((30\text{ mi})(5280\text{ ft/mi}))$

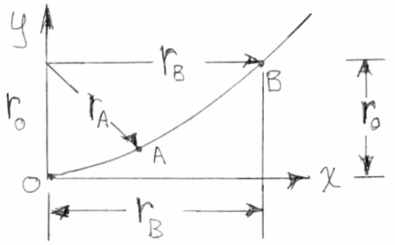
$= 21.0672 \times 10^6\text{ ft}$

$v = 36{,}557\text{ ft/s}$ $\qquad v = 6.92\text{ mi/s}$ ◀

(*c*) For $r_2 = 20.9088 \times 10^6\text{ ft/s}$

$v = 36{,}695\text{ ft/s}$ $\qquad v = 6.95\text{ mi/s}$ ◀

Chapter 13, Solution 37.



$$r_0 = 620\text{ mi} + 3960\text{ mi} = 4580\text{ mi} = 24{,}182{,}400\text{ ft}$$

$$r_B = 3960\text{ mi} + 5200\text{ mi} = 9160\text{ mi} = 48{,}364{,}800\text{ ft}$$

Parabola: $y = Kx^2$

At B: $$r_0 = Kr_B^2 \qquad 24{,}182{,}400\text{ ft} = K\left(48{,}364{,}800\text{ ft}\right)^2$$

$$K = 10.3381 \times 10^{-9}\text{ ft}^{-1}$$

At A: $$x_A = r_A\left(\sin 45°\right),\ y_A = Kx_A^2 = r_0 - r_A\left(\cos 45°\right)$$

$$r_0 - r_A\left(\cos 45°\right) = Kr_A^2\left(\sin 45°\right)^2$$

$$Kx_A^2 + x_A - r_0 = 0$$

$$v_0 = (6.5)(5280) = 34{,}320\text{ ft/s}$$

$$x_A = \frac{1}{2K}\left(-1 + \sqrt{1 + 4Kr_0}\right)$$

$$x_A = 20.0334 \times 10^6\text{ ft},\ r_A = 14.1657 \times 10^6\text{ ft.}$$

(*a*) $$U_{0\to A} = \frac{GM\cancel{m}}{r_A} - \frac{GM\cancel{m}}{r_0} = \frac{1}{2}\cancel{m}v_A^2 - \frac{1}{2}\cancel{m}v_0^2$$

$$GM = 32.2\left(20.9088 \times 10^6\right)^2,\ v_A^2 = v_0^2 + 2GM\left(\frac{1}{r_A} - \frac{1}{r_0}\right)$$

$$v_A^2 = \left(34320\right)^2 + 2GM\left(\frac{1}{14.1657 \times 10^6} - \frac{1}{24.1824 \times 10^6}\right)$$

$v_A = 44734$ ft/s $\qquad v_A = 8.47$ mi/s ◀

(*b*) $$v_B^2 = v_0^2 + 2GM\left(\frac{1}{r_B} - \frac{1}{r_0}\right)$$

$$v_B^2 = \left(34320\right)^2 + 2GM\left(\frac{1}{48.3648 \times 10^6} - \frac{1}{28.1824 \times 10^6}\right)$$

$v_B = 24408$ ft/s $\qquad v_B = 4.62$ mi/s ◀

Chapter 13, Solution 38.

(*a*) $g = 1.62 \text{ m/s}^2$ Use work and energy $T_1 + U_{1\to 2} = T_2$ (1)

where $$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m(600)^2 = 180{,}000\ m$$

$$T_2 = 0 \text{ (maximum elevation)}$$

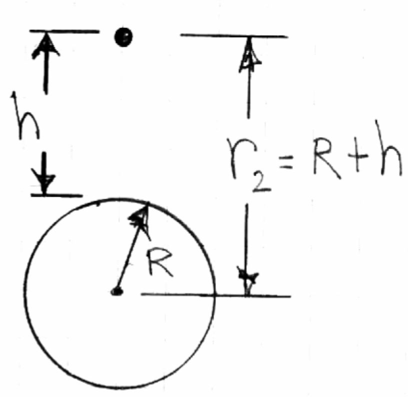
$$U_{1\to 2} = -mgh$$

$$= -\ m(1.62)h = -1.62mh$$

Substituting into (1) $180{,}000\ \cancel{m} - 1.62\ \cancel{m}h = 0$

$$h = 111.11\times 10^3 \text{ m}$$

$h = 111.1$ km ◀



(*b*) $F = \dfrac{GMm}{R^2}$ so $T_1 = 180{,}000\ m$ (same as above)

$$W = \frac{GMm}{R^2} = mg \Rightarrow GM = gR^2$$

At some elevation r $F = -\dfrac{GMm}{r^2}$

so,

$$U_{1\to 2} = \int_R^{R+h} -\frac{GMm}{r^2}dr$$

$$= GMm\left(\frac{1}{r}\right)\Bigg|_R^{r_2} = gR^2m\left(\frac{1}{r_2} - \frac{1}{R}\right)$$

$$= (1.62)(1.740\times 10^6)^2\, m\left(\frac{1}{\mathrm{r}_2} - \frac{1}{1.740\times 10^6}\right)$$

Substituting into (1)

$$180,000\,m + 4.9047\times 10^{12}\ \text{m}\left[\frac{1}{r_2} - \frac{1}{1.74\times 10^{6}}\right] = 0$$

Solve for r_2

$$r_2 = 1.8587\times 10^{6}\ \text{m}$$

$$= 1858.7\ \text{km}$$

so, $h = r_2 - R = 1858.7 - 1740$

$$= 118.7\ \text{km}$$

$h = 118.7$ km ◀

Chapter 13, Solution 39.

Assume the blocks move as one: $T_2 - \cancelto{0}{T_1} = U_{1-2}$

$$\frac{1}{2}(m_A + m_B)v^2 = \frac{1}{2}kx^2 - U_{1-2\,\text{friction}}$$

$$\frac{1}{2}(3\text{ kg})v^2 = \frac{1}{2}(180\text{ N/m})(0.1\text{ m})^2 - (3)(9.81)(0.1)(0.1\text{ m})$$

$$v^2 = 0.4038 \qquad v = 0.63545\text{ m/s}$$

Check assumption at release, $18\text{ N} > \mu_s(3)(9.81) = 4.41\text{ N}$

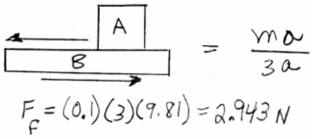
$\therefore$ Slips at the horizontal surface

At release

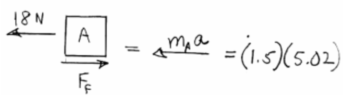
$F_s = 18\text{ N}$

$\overset{+}{\leftarrow}\ \Sigma F_x = ma_x$

$18 - 2.94 = 3a \qquad a = 5.02\text{ m/s}^2$



For *A* alone:



$\overset{+}{\leftarrow}\ \Sigma F_x = ma_x$

$18 - F_f = (1.5)(5.02)$

$F_f = 10.47\text{ N}$

$$F_f = 18 - 7.53 = 10.47\text{ N} < \mu_s(1.5)(9.81) = (0.95)(14.175) = 13.98\text{ N}$$

$\therefore$ *A* and *B* move as one, thus (*a*) $\qquad v = 0.635\text{ m/s}$ ◀

(*b*) $\qquad v_{\max}$ is max at $a = 0$,

$$F_s = F_f = 0 = 180x - 2.943, \qquad x = 0.01635\text{ m}$$

$$\frac{1}{2}(m_A + m_B)v_{\max}^2 = \frac{1}{2}k(x_0^2 - x^2) - F_f(x_0 - x)$$

$$\frac{1}{2}(3)v_{\max}^2 = \frac{1}{2}(180)[(0.1)^2 - (0.01635)^2] - 2.943(0.1 - 0.01635)$$

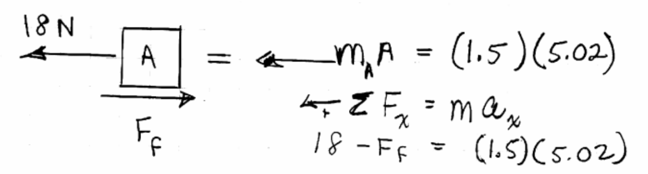
$v_{\max} = 0.648\text{ m/s}$ ◀

Chapter 13, Solution 40.

From problem 13.39 assuming the blocks move together,

$a = 5.02\ \text{m/s}^2$ at release.

$W_A = W_B = (1.5\ \text{kg})(9.81\ \text{m/s}^2) = 14.715\ \text{N}$



$$= \xleftarrow{\;} m_A A = (1.5)(5.02)$$

$$\overset{+}{\leftarrow} \Sigma F_x = m a_x$$

$$18 - F_f = (1.5)(5.02)$$

$F_f = 18 - 7.53 = 10.47\ \text{N} < \mu_s(1.5)(9.81) = 0.35(14.715) = 5.15\ \text{N}$

$\therefore$ Block *A* slides on Block *B*

A alone:

(*a*) $$T_2 - \overset{0}{\cancel{T_1}} = U_{1-2}, \quad \frac{1}{2}(m_A)v^2 = \frac{1}{2}kx - U_{1-2\ \text{friction}}$$

$$\frac{1}{2}(1.5)(v^2 = \frac{1}{2}(180\ \text{N/m})(0.1\ \text{m}^2) - 14.175(0.3)(0.1\ \text{m})$$

$$v^2 = 0.6114$$

$v = 0.782\ \text{m/s}$ ◀

(*b*) $v = v_{\max}$ at acceleration = 0,

$$F_s - F_f = 0 = kx - \mu_k W_A$$

$$180x = (0.30)(14.715) = 4.4145, \quad x = 0.0245\ \text{m}$$

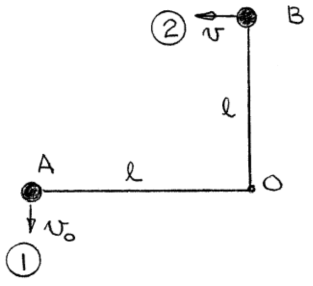
$$T_2 - \overset{0}{\cancel{T_1}} = \frac{1}{2}k(x_0^2 - x^2) - W_A\mu_k(x_0 - x)$$

$$\frac{1}{2}(1.5)v_{\max}^2 = 0.84598 - 0.33329 = 0.51268$$

$v_{\max} = 0.827\ \text{m/s}$ ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 41.



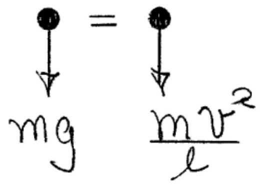
$$T_1 = \frac{1}{2}mv_0^2$$

$$T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = -mgl$$

$$T_1 + U_{1-2} = T_2 \qquad \frac{1}{2}mv_0^2 - mgl = \frac{1}{2}mv^2$$

$$v_0^2 = v^2 + 2gl$$



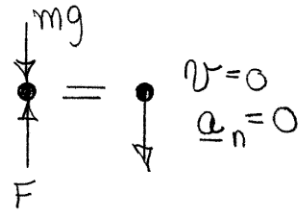
Newtons' law at ②

(*a*) For minimum v, tension in the cord must be zero.

Thus $$v^2 = gl$$

$$v_0^2 = v^2 + 2gl = 3gl$$

$$v_0 = \sqrt{3gl} \blacktriangleleft$$

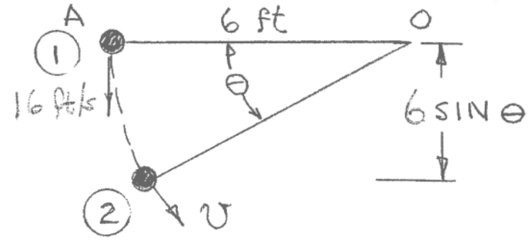


(*b*) Force in the rod can support the weight so that v can be zero.

Thus $$v_0^2 = 0 + 2gl$$

$$v_0 = \sqrt{2gl} \blacktriangleleft$$

Chapter 13, Solution 42.



$$T_1 = \frac{1}{2}mv_0^2 = \frac{1}{2}m(16)^2 = 128\text{ m}$$

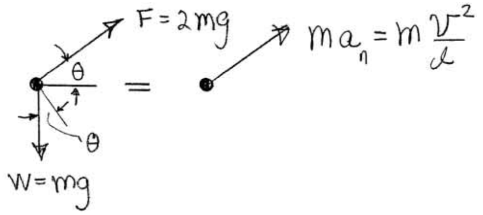
$$T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = mg(6\sin\theta)$$

$$T_1 + U_{1-2} = T_2:\quad 128m + 6mg\sin\theta = \frac{1}{2}mv^2$$

$$256 + 12g\sin\theta = v^2 \qquad (a)$$

Newton's law



$$+\nearrow\ \Sigma F = ma_n:\quad 2mg - mg\sin\theta = \frac{mv^2}{6}$$

Using (*a*)

$$g(12 - 6\sin\theta) = 256 + 12g\sin\theta$$

$$18g\sin\theta = 12g - 256$$

$$\sin\theta = \frac{12(32.2) - 256}{18(32.2)} = 0.22498$$

$\theta = 13.00°$ ◀

Chapter 13, Solution 43.

Use work - energy : position 1 is bottom and position 2 is the top

$$T_1 + U_{1\to 2} = T_2 \qquad (1)$$

where,

$$T_1 = \frac{1}{2}mv_0^2$$

$$T_2 = \frac{1}{2}mv_2^2$$

$$U_{1\to 2} = -mgh = -mg(0.5)$$

Substituting into (1)

$$\frac{1}{2}\cancel{m}v_0^2 - (0.5)\cancel{m}g = \frac{1}{2}\cancel{m}v_2^2$$

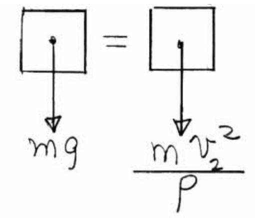
so

$$v_0^2 = v_2^2 + g \qquad (2)$$

(a) Slender rod $v_2 = 0 \Rightarrow v_0 = \sqrt{g}$

$v_0 = 3.13$ m/s ◀

(b) Cord, so the critical condition is tension = 0 at the top



$$\downarrow \Sigma F_n = ma \Rightarrow mg = \frac{mv_2^2}{\rho} \Rightarrow v_2^2 = g\rho$$

Substituting into (2)

$$v_0^2 = g\rho + g = 9.81(0.25 + 1)$$

$$= 12.2625$$

$$v_0 = 3.502 \text{ m/s}$$

$v_0 = 3.50$ m/s ◀

Chapter 13, Solution 44.

Use work - energy : position 1 is at A, position 2 is at B.

$$T_1 + U_{1\to 2} = T_2 \qquad (1)$$

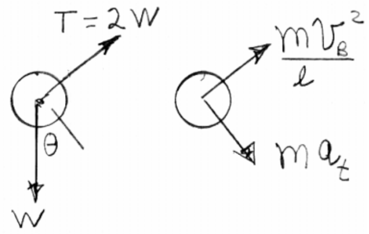
Where $T_1 = 0$; $U_{1\to 2} = mgl\sin\theta$; $T_2 = \frac{1}{2}mv_B^2$

Substitute

$$0 + mgl\sin\theta = \frac{1}{2}mv_B^2$$

$$v_B^2 = 2gl\sin\theta \qquad (2)$$

For $T = 2W$ use Newtons 2nd law.



$$\nearrow \Sigma F_n = ma_n \Rightarrow 2W - W\sin\theta = \frac{mv_B^2}{l} \qquad (3)$$

Substitute (2) into (3)

$$2\cancel{m}g - \cancel{m}g\sin\theta = 2\cancel{m}g\frac{\cancel{l}\sin\theta}{\cancel{l}}$$

$$2 = 3\sin\theta$$

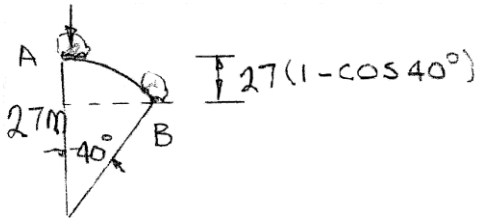
$$\text{or } \sin\theta = \frac{2}{3} \Rightarrow \theta = 41.81°$$

$\theta = 41.8°$ ◀

Chapter 13, Solution 45.

$$v_A = 0 \qquad T_A = 0 \qquad T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(250\text{ kg})v_B^2 = 125v_B^2$$

$$W = (1250\text{ kg}) \times \left(9.81\text{ m/s}^2\right)$$



$$U_{A-B} = W(27)(1 - \cos 40°)$$

$$U_{A-B} = \left(250\text{ kg} \times 9.81\text{ m/s}^2\right)(27\text{ m})(0.234)$$

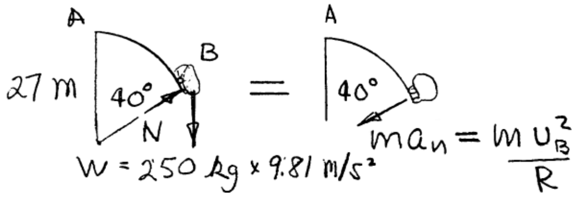
$$U_{A-B} = 15495\text{ J}$$

$$T_A + U_{A-B} = T_A \qquad 0 + 15495 = 125v_B^2$$

$$v_B^2 = \frac{(15495\text{ J})}{(125\text{ kg})}$$

$$v_B^2 = 124.0\text{ m}^2/\text{s}^2$$

Newtons Law at B



$$+\nearrow \; N - W\cos 40° = \frac{-mv_B^2}{R};\; v_B^2 = 124.0\text{ m}^2/\text{s}^2$$

$$N = \left(250\text{ kg} \times 9.81\text{ m/s}^2\right)(\cos 40°) - \frac{(250\text{ kg})\left(124.0\text{ m}^2/\text{s}^2\right)}{27\text{ m}}$$

$$N = 1879 - 1148 = 731\text{ N}$$

$N = 731\text{ N}$ ◀

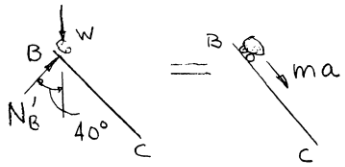
Chapter 13, Solution 46.

Normal force at B

See solution to Problem 13.45, $N_B = 731.0 \text{ N}$

Newtons Law

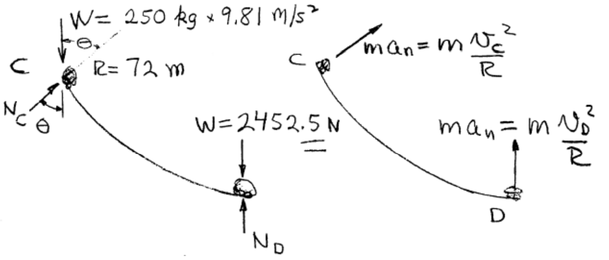
From B to C (car moves in a straight line)


W
B
N_B'
40°
C
=
B
ma
C

$$+\nearrow \; N_B' - W\cos 40° = 0$$

$$N_B' = \left(250 \text{ kg} \times 9.81 \text{ m/s}^2\right)\cos 40° = 1878.7 \text{ N}$$

At C and D (car in the curve at *C*)


W= 250 kg × 9.81 m/s²
θ
C
R= 72 m
N_C
θ
W=2452.5 N
N_D
=
C
$ma_n = m\frac{v_C^2}{R}$
$ma_n = m\frac{v_D^2}{R}$
D

At C

$$+\nearrow \; N_C - W\cos\theta = \frac{W}{g}\frac{v_C^2}{R}$$

$$N_C = \left(250 \text{ kg} \times 9.81 \text{ m/s}^2\right)\left(\cos\theta + \frac{v_C^2}{gR}\right)$$

At D

$$+\uparrow \; N_D - W = +\frac{W}{g}\frac{v_D^2}{R}$$

continued

$$N_D = (250 \times 9.81)\left(1 + \frac{v_D^2}{gR}\right)$$

Since $v_D > v_C$ and $\cos\theta < 1,\ N_D > N_C$

Work and energy from A to D

$$v_A = 0,\ T_A = 0 \qquad T_D = \frac{1}{2}mv_D^2 = 125v_D^2$$

$$U_{A-D} = W(27 + 18) = (250\text{ kg})\left(9.81\text{ m/s}^2\right)(45\text{ m}) = 110362.5$$

$$T_A + U_{A-D} = T_D \qquad 0 + 110362.5 = 125v_D^2$$

$$v_D^2 = 882.90$$

$$N_D = 250g\left(1 + \frac{v_D^2}{72g}\right) = 250(9.81)\left(1 + \frac{882.90}{72(9.81)}\right) = 5518.1\text{ N}$$

$$N_{\min} = N_B = 731\text{ N};\ N_{\max} = N_D = 5520\text{ N} \blacktriangleleft$$

Chapter 13, Solution 47.

Kinematics: $a = \text{constant}, \quad v = at, \quad x = \frac{1}{2}at^2$

$$x = 110 = \frac{1}{2}a(5.4)^2, \quad a = 7.5446 \text{ ft/s}^2$$

$$F = \text{constant} = ma = \left(\frac{150 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(7.5446 \text{ ft/s}^2\right) = 35.1456 \text{ lb}$$

$$v = 7.5446\, t$$

$$\text{Power} = Fv = mav = \left(\frac{150}{32.2}\right)(7.5446)^2 t$$

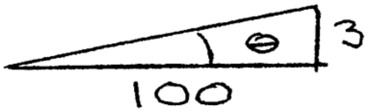
$$\text{Average power} = \frac{1}{5.4}\int_0^{5.4} Fv\, dt = \frac{(150/32.2)(7.5446)^2}{5.4}\left(\frac{(5.4)^2}{2} - 0\right)$$

$$\text{Average power} = 715.93 \text{ ft}\cdot\text{lb/s}$$

$$\text{Average power} = 1.302 \text{ hp} \blacktriangleleft$$

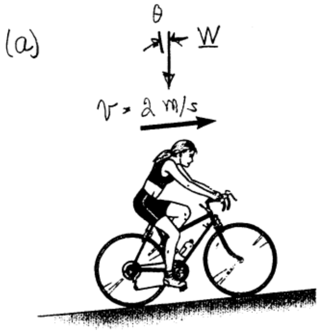
Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 48.



$$\tan\theta = \frac{3}{100}$$

$$\theta = 1.718°$$



$$W = W_B + W_W = (7 + 60)\,\text{kg} \times 9.81\,\text{m/s}^2$$

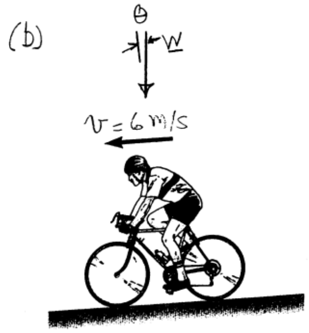
$$W = 657.3\ \text{N}$$

$$P_W = W \cdot v = (W\sin\theta)(v)$$

$$P_W = (657.3)(\sin 1.718°)(2)$$

$$P_W = 39.41\ \text{W}$$

$P_W = 39.4$ W ◀



$$W = W_B + W_m = (9 + 90)\,\text{kg} \times 9.81\,\text{m/s}^2$$

$$W = 971.2\ \text{N}$$

Brake must dissipate the power generated by the bike and the man going down the slope at 6 m/s.

$$P_B = W \cdot v = (W\sin\theta)(v)$$

$$P_B = (971.2)(\sin 1.718°)(6) = 174.701$$

$P_B = 174.7$ W ◀

Chapter 13, Solution 49.

(*a*) $\left(P_p\right)_A = (F)(v_A) = (W_A + W_L)(v_A) = (2800 + 650)v_A$

$$v_A = \text{s/t} = \frac{6.5\ \text{ft}}{16\ \text{s}} = 0.40625\ \text{ft/s}$$

$$\left(P_p\right)_A = (3450\ \text{lb})(0.40625\ \text{ft/s}) = 1401.56\ \text{lb}\cdot\text{ft/s}$$

$$1\ \text{hp} = 550\ \text{ft}\cdot\text{lb/s},\ \left(P_p\right)_A = 2.548\ \text{hp}$$

$$\left(P_p\right)_A = 2.55\ \text{hp} \blacktriangleleft$$

(*b*)

$$\left(P_E\right)_A = \frac{\left(P_p\right)_A}{\eta} = \frac{2.55}{0.82}$$

$$\left(P_E\right)_A = 3.11\ \text{hp} \blacktriangleleft$$

Chapter 13, Solution 50.

(*a*) Material is lifted to a height b at a rate, $\left(m\ \text{kg/h}\right)\left(g\ \text{m/s}^2\right) = \left[mg\left(\text{N/h}\right)\right]$

Thus,

$$\frac{\Delta U}{\Delta t} = \frac{\left[mg\left(\text{N/h}\right)\right]\left[b\left(m\right)\right]}{\left(3600\ \text{s/h}\right)} = \left(\frac{mgb}{3600}\right)\text{N}\cdot\text{m/s}$$

$$1000\ \text{N}\cdot\text{m/s} = 1\,\text{kw}$$

Thus, including motor efficiency, η

$$P\left(\text{kw}\right) = \frac{mgb\left(\text{N}\cdot\text{m/s}\right)}{\left(3600\right)\left(\dfrac{1000\ \text{N}\cdot\text{m/s}}{\text{kw}}\right)\left(\eta\right)}$$

$$P\left(\text{kw}\right) = 0.278\times10^{-6}\,\frac{mgb}{\eta} \blacktriangleleft$$

(*b*)

$$\frac{\Delta U}{\Delta t} = \frac{\left[W\left(\text{tons/h}\right)\left(2000\ \text{lb/ton}\right)\right]\left[b\left(\text{ft}\right)\right]}{3600\ \text{s/h}}$$

$$= \frac{Wb}{1.8}\ \text{ft}\cdot\text{lb/s};\ 1\text{hp} = 550\ \text{ft}\cdot\text{lb/s}$$

With η,

$$hp = \left[\frac{Wb}{1.8}\left(\text{ft}\cdot\text{lb/s}\right)\right]\left[\frac{1\text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right]\left[\frac{1}{\eta}\right]$$

$$hp = \frac{1.010\times10^{-3}Wb}{\eta} \blacktriangleleft$$

Chapter 13, Solution 51.

For constant power, P:

$$P = Fv = mav = m\frac{dv}{dt}v$$

Separate variables

$$\int_0^t dt = \int_0^{v_1} \frac{m}{P}\frac{dv}{v} \Rightarrow t = \frac{m}{P}\left(\frac{v_1^2}{2} - \frac{v_0^2}{2}\right) \tag{1}$$

Distance

$$P = mv\frac{dv}{dx}\frac{dx}{dt} = mv^2\frac{dv}{dx}$$

Separate variables

$$\int_0^x dx = \frac{m}{P}\int_{v_0}^{v_1} v^2 dv \Rightarrow x = \frac{m}{P}\left(\frac{v_1^3}{3} - \frac{v_0^3}{3}\right) \tag{2}$$

with numbers

(a) $v_0 = 36 \text{ km/h} = 10 \text{ m/s}; \;\; v_1 = 54 \text{ km/h} = 15 \text{ m/s},$ so,

$$t = \frac{15\times10^3 \text{ kg}}{2\left(50\times10^3 \text{ W}\right)}\left[\left(15 \text{ m/s}\right)^2 - \left(10 \text{ m/s}\right)^2\right] \qquad \Rightarrow t = 18.75 \text{ s} \blacktriangleleft$$

$$x = \frac{15\times10^3}{(3)\left(50\times10^3\right)}\left[15^3 - 10^3\right] = 237.5 \text{ m} \qquad \Rightarrow x = 238 \text{ m} \blacktriangleleft$$

(b) $v_0 = 54 \text{ km/h} = 15 \text{ m/s}; \;\; v_1 = 72 \text{ km/h} = 20 \text{ m/s}$

$$t = \frac{15\times10^3}{(2)\left(50\times10^3\right)}\left[20^2 - 15^2\right] = 26.25 \text{ s} \qquad \Rightarrow t = 26.2 \text{ s} \blacktriangleleft$$

$$x = \frac{15\times10^3}{(3)\left(50\times10^3\right)}\left[20^3 - 15^3\right] = 462.5 \text{ m} \qquad \Rightarrow x = 462 \text{ m} \blacktriangleleft$$

Chapter 13, Solution 52.

$$(a)\quad P = F\cdot v = \left(m\frac{dv}{dt}\right)v = \text{constant}$$

$$\therefore\ m\int_{2.0}^{4.3} v\,dv = P\int_0^5 dt$$

$$60\text{ kg}\left(\frac{(4.3\text{ m/s})^2 - (20\text{ m/s})^2}{2}\right) = 5\,P,\quad P = 86.94\text{ W}$$

$P = 86.9$ W ◀

$$(b)\quad P = F\cdot v = \left(mv\frac{dv}{dx}\right)v = \text{constant}$$

$$\therefore\ m\int_{2.0}^{4.3} v^2 dv = P\int_0^x dx$$

$$60\left(\frac{(4.3)^3 - (2.0)^3}{3}\right) = 86.94\,x$$

$x = 16.45$ m ◀

Chapter 13, Solution 53.

Motion is determined as a function of time as,

$$x = 12000 \ln\left(\cosh 0.03t\right)$$

Velocity $$v = \frac{dx}{dt} = 12000\left(\frac{1}{\cosh 0.03t}\right)\left(\sinh 0.03t\right)\left(0.03\right)$$

$$v = \frac{360 \sinh 0.03t}{\cosh 0.03t}$$

Power dissipated $$P = Dv = \left(0.01v^2\right)v = 0.01v^3$$

$$P = 0.01\left(360\right)^3\left[\frac{\sinh 0.03t}{\cosh 0.03t}\right]^3 = 466.56\times10^3\left[\frac{e^{0.03t} - e^{-0.03t}}{e^{0.03t} + e^{-0.03t}}\right]^3$$

(a) $t = 10$ s, $P = 11534$ ft·lb/s $P = 21.0$ hp ◀

(b) $t = 15$ s, $P = 35037$ ft·lb/s $P = 63.7$ hp ◀

Chapter 13, Solution 54.

Motion is defined by the following function:

$$a = 11\, e^{-0.0005x} = v\frac{dv}{dx}$$

$$\int_0^v v dv = 11\int_0^x e^{-0.0005x}\ dx = \frac{-11}{0.0005}\int_0^{-0.0005x} e^u\ du$$

$$\frac{v^2}{2} = 22000\left(e^{-0.0005x} - 1\right)$$

$$v^2 = 44000\left(1 - e^{-0.0005x}\right)$$

$$v = 209.76\left(1 - e^{-0.0005x}\right)^{\frac{1}{2}}$$

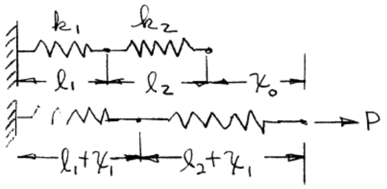
Power dissipated $P = Dv = 0.01v^3$

$$P = 92295\left[1 - e^{-0.0005x}\right]^{\frac{3}{2}}$$

(a) $x = 600$ ft , $P = 12178$ ft·lb/s $P = 22.1$ hp ◀

(b) $x = 1200$ ft, $P = 27971$ ft·lb/s $P = 50.9$ hp ◀

Chapter 13, Solution 55.



System is in equilibrium in deflected x_0 position.

Case (*a*) Force in both springs is the same $= P$

$$x_0 = x_1 + x_2$$

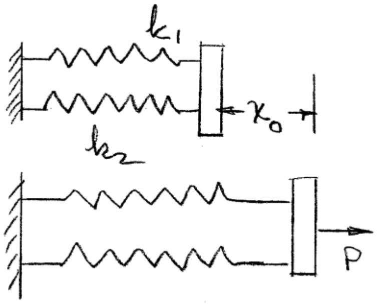
$$x_0 = \frac{P}{k_e}$$

$$x_1 = \frac{P}{k_1} \qquad x_2 = \frac{P}{k_2}$$

Thus
$$\frac{P}{k_e} = \frac{P}{k_1} + \frac{P}{k_2}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_e = \frac{k_1 k_2}{k_1 + k_2} \blacktriangleleft$$



Case (*b*) Deflection in both springs is the same $= x_0$

$$P = k_1 x_0 + k_2 x_0$$

$$P = \left(k_1 + k_2\right) x_0$$

$$P = k_e x_0$$

Equating the two expressions for

$$P = \left(k_1 + k_2\right) x_0 = k_e x_0$$

$$k_e = k_1 + k_2 \blacktriangleleft$$

Chapter 13, Solution 56.

Use conservation of energy

Let position 1 be where A is compressed 0.1 m; position 2 when B is compressed a maximum distance

So

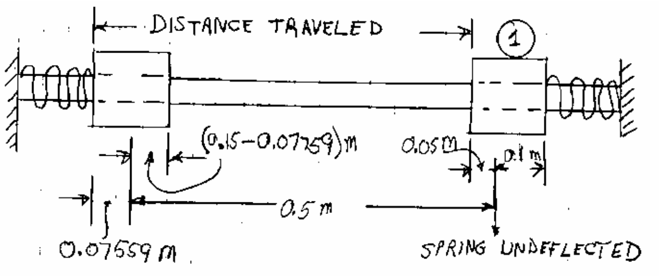
$$T_1 + V_1 = T_2 + V_2 \qquad (1)$$

Where $$T_1 = 0; \quad V_1 = \frac{1}{2}k_A x_1^2 = \frac{1}{2}(1600 \text{ N/m})(0.1 \text{ m})^2 = 8 \text{ J}$$

$$T_2 = 0; \quad V_2 = \frac{1}{2}k_B x_2^2 = \frac{1}{2}(2800 \text{ N/m})x_2^2 = 1400x_2^2$$

Substituting into (1)

$$0 + 8 = 0 + 1400\, x_2^2 \;\Rightarrow\; x_2 = 0.07559 \text{ m}$$



This answer is independent of mass

$$\text{Distance traveled} = 0.5 \text{ m} - 0.05 \text{ m} + 0.07559 \text{ m} = 0.526 \text{ m}$$

The maximum velocity will occur when the mass is between the two springs

where $$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0; \qquad V_1 = 8 \text{ J (same as before)}$$

$$T_2 = \frac{1}{2}mv_{\max}^2; \qquad V_2 = 0$$

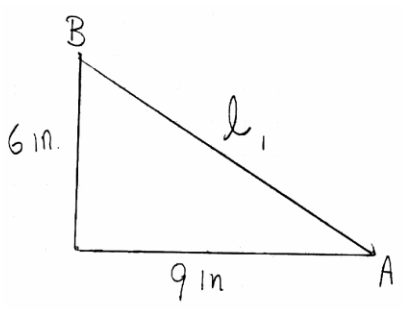
Substituting into (1)

$$0 + 8 = \frac{1}{2}mv_{\max}^2 + 0; \quad v_{\max}^2 = \frac{16}{m}$$

For $m = 1$ kg $\qquad v_{\max}^2 = 16 \qquad$ (*a*) $\qquad v_{\max} = 4$ m/s ◀

For $m = 2.5$ kg $\qquad v_{\max}^2 = \dfrac{16}{2.5} = 6.4 \qquad$ (*b*) $\qquad v_{\max} = 2.53$ m/s ◀

Chapter 13, Solution 57.

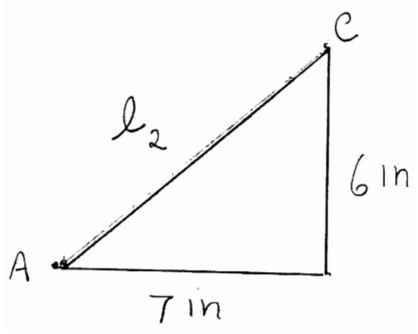


$$\ell_1 = \sqrt{6^2 + 9^2} = 10.817 \text{ in.}$$

$$\ell_0 = \sqrt{(6)^2 + (8)^2} = 10 \text{ in.} = 0.8333 \text{ ft}$$

Stretch $= 10.817 - 10 = 0.817$ in.

$$S_1 = 0.06805 \text{ ft}$$



$$\ell_2 = \sqrt{(7)^2 + (6)^2} = 9.215 \text{ in.}$$

Stretch $= 9.2195 - 10 = -0.7805$ in.

$$S_2 = 0.06504 \text{ ft}$$

$$T_1 = 0, \ \ V_2 = 0$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\left(\frac{4}{32.2}\right)v_2^2$$

$$V_1 = \frac{1}{2}(33{,}600 \text{ lb/ft})\left(S_1^2 + S_2^2\right)$$

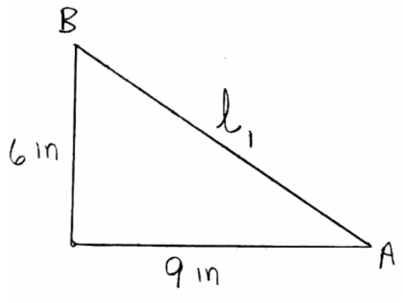
$$V_1 = (16{,}800)(0.008861) = 148.86 \text{ ft}\cdot\text{lb}$$

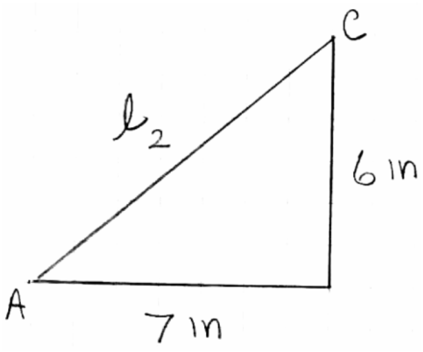
$$\cancelto{0}{T_1} + V_1 = T_2 + \cancelto{0}{V_2}$$

$$v_2^2 = 2396.7$$

$v_2 = 49.0$ ft/s ◀

Chapter 13, Solution 58.





$$k = 2800 \text{ lb/in.} = 33,600 \text{ lb/ft}$$

$$T_1 = 0 \quad V_1 = \frac{1}{2}k\left(\Delta l_1\right)^2 = \frac{1}{2}(33,600)\left(\frac{1}{12}\right)^2$$

$$= 116.667 \text{ ft} \cdot \text{lb}$$

$$\ell_1 = \sqrt{6^2 + 9^2} = 10.817 \text{ in.} = 0.9014 \text{ ft}$$

$$S_1 = \text{Stretch} = 10.817 - 10 = 0.817 \text{ in.} = 0.06808 \text{ ft}$$

$$\ell_2 = \sqrt{6^2 + 7^2} = 9.2195 \text{ in.}$$

$$S_2 = \text{Stretch} = 9.2195 - 9 = 0.2195 \text{ in.} = 0.018295 \text{ ft}$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\left(\frac{4}{32.2}\right)v_2^2 = 0.0621\, v_2^2$$

$$V_2 = \frac{1}{2}(33,600)\left(S_1^2 + S_2^2\right)$$

$$= 16800\left[\left(0.06808\right)^2 + \left(0.018295\right)^2\right]$$

$$= 83.489 \text{ ft} \cdot \text{lb}$$

$$\cancelto{0}{T_1} + V_1 = T_2 + V_2$$

$$116.667 = 0.06211\, v_2^2 + 83.489$$

$$v_2 = 23.1 \text{ ft/s} \blacktriangleleft$$

Chapter 13, Solution 59.

Use conservation of energy $T_1 + V_1 = T_2 + V_2$ (1)

(*a*) Position 1 is at *A* and position 2 is at *B*

$T_1 = 0;$ $V_1 = \frac{1}{2}kx_1^2$ where $x_1 = \ell - \ell_0$

$$\ell = \left[500^2 + 400^2 + 350^2\right]^{\frac{1}{2}} = 729.726 \text{ mm}$$

So $\ell - \ell_0 = 429.726$ mm

$$V_1 = \frac{1}{2}(1500 \text{ N/m})(0.429726 \text{ m})^2 = 13.8498 \text{ J}$$

At *B* $\ell = \left[350^2 + 400^2\right]^{\frac{1}{2}} = 531.507 \text{ mm} \Rightarrow \ell - \ell_0 = 231.507 \text{ mm}$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.75)v_2^2 = 0.375\ v_2^2$$

$$V_2 = \frac{1}{2}(150)(0.231507)^2 = 4.01966 \text{ J}$$

Substituting into (1) $0 + 13.8498 = 0.375\ v_2^2 + 4.01966$

$v_2 = v_B = 5.12$ m/s ◀

(*b*) At *E* $T_1 = 0;\ V_1 = 13.8498$ (same as before)

At *E* $\ell = \left[350^2 + 500^2\right]^{\frac{1}{2}} = 610.328 \text{ mm} \Rightarrow \ell - \ell_0 = 310.328 \text{ mm}$

$$T_3 = \frac{1}{2}mv_E^2 = 0.375\ v_E^2$$

$$v_E = \frac{1}{2}kx^2 = \frac{1}{2}(150)(0.310328)^2 = 7.223 \text{ J}$$

Substituting into (1)

$0 + 13.8498 = 0.375\ v_E^2 + 7.223 \Rightarrow v_E = 4.20$ m/s ◀

Chapter 13, Solution 60.

Conservation of energy $T_1 + V_1 = T_2 + V_2$

At A $\ell = \left[500^2 + 400^2 + 350^2\right]^{\frac{1}{2}} = 729.726 \text{ mm} \Rightarrow \ell - \ell_0 = 729.726 - 450 = 279.726 \text{ mm}$

So $T_A = 0; \quad V_A = \frac{1}{2}kx^2 = \frac{1}{2}(150)(0.279726)^2 = 5.8685 \text{ J}$

At B $\ell = \left[350^2 + 400^2\right]^{\frac{1}{2}} = 531.507 \Rightarrow \ell - \ell_0 = 81.507 \text{ mm}$

$T_B = \frac{1}{2}mv_B^2 = 0.375\, v_B^2; \quad V_B = \frac{1}{2}kx^2 = \frac{1}{2}(150)(0.081507)^2 = 0.49825 \text{ J}$

Substituting into (1) $0 + 5.8685 = 0.375\, v_B^2 + 0.49825$

$v_B = 3.78 \text{ m/s}$ ◀

At E $\ell = \left[(350)^2 + (500)^2\right]^{\frac{1}{2}} = 610.328 \text{ mm} \Rightarrow \ell - \ell_0 = 160.328 \text{ mm}$

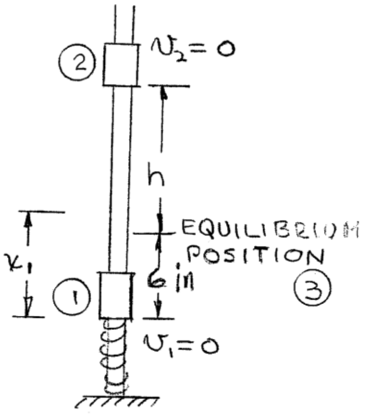
So $T_E = \frac{1}{2}mv_E^2 = 0.375\, v_E^2; \quad V_E = \frac{1}{2}kx^2 = \frac{1}{2}(150)(0.160328)^2 = 1.9279 \text{ J}$

Substituting into (1) $0 + 5.8685 = 0.375\, V_E^2 + 1.9279$

$v_E = 3.24 \text{ m/s}$ ◀

The fact the cord becomes slack doesn’t matter.

Chapter 13, Solution 61.



(*a*) Maximum height when $v_2 = 0$

$$\therefore\ T_1 = T_2 = 0$$

$$V = V_g + V_e$$

Position ① $\left(V_g\right)_1 = 0$

$$x_1 = \frac{6\text{ lb}}{15\text{ lb/in.}} + 6\text{ in.} = 0.4 + 6 = 6.4\text{ in.}$$

$$\left(V_e\right)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(15\text{ lb/in.})(6.4\text{ in.})^2$$

$$= 307.2\text{ lb}\cdot\text{in.} = 25.6\text{ lb}\cdot\text{ft}$$

Position ② $\left(V_g\right)_2 = mg\left(\frac{6}{12} + h\right) = 6(0.5 + h)$

$$\left(V_e\right)_2 = 0$$

$$T_1 + V_1 = T_2 + V_2:\ \left(V_g\right)_1 + \left(V_e\right)_1 = \left(V_g\right)_2 + \left(V_e\right)_2$$

$$25.6 = 6(0.5 + h)$$

$h = 3.767$ ft $\qquad h = 45.2$ in. ◀

(*b*) Maximum velocity occurs when acceleration is 0, equilibrium position

$$T_3 = \frac{1}{2}mv_3^2 = \frac{1}{2}\left(\frac{6}{32.2}\right)v_3^2 = 0.093167v_3^2$$

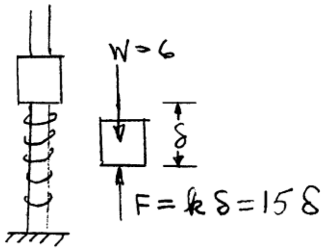
$$V_3 = \left(V_g\right)_3 + \left(V_e\right)_3 = 6(6) + \frac{1}{2}k\left(x_1 - 6\right)^2 = 36 + 7.5(6.4 - 6)^2$$

$$= 37.2\text{ lb}\cdot\text{in.} = 3.1\text{ lb}\cdot\text{ft}$$

$$T_1 + V_1 = T_3 + V_3:\quad 25.6 = 0.093167v_3^2 + 3.1$$

$v_{\max} = 15.54$ ft/s ◀

Chapter 13, Solution 62.

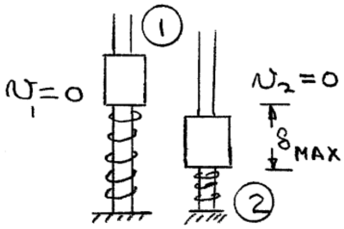


(*a*) Collar is in equilibrium.

$$+\uparrow \Sigma F = (15 \text{ lb/in.})\delta - 6 \text{ lb}$$

$$\delta = \frac{(6 \text{ lb})}{(15 \text{ lb/in.})} = 0.4 \text{ in.}$$

$$\delta_{\max} = 0.4 \text{ in.} ◀$$



(*b*) Maximum compression occurs when velocity at ② is zero.

$$T_1 = 0 \qquad V_1 = 0$$

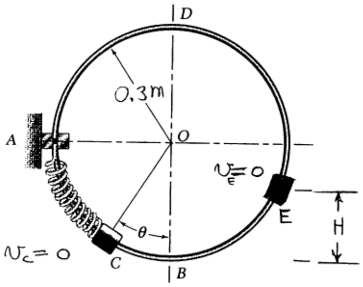
$$T_2 = 0 \qquad V_2 = -W\delta_{\max} + \frac{1}{2}k\delta_{\max}^2$$

$$W = \frac{1}{2}k\delta_{\max}$$

$$\delta_{\max} = \frac{2(6 \text{ lb})}{(15 \text{ lb/in.})} = 0.8 \text{ in.}$$

$$\delta_{\max} = 0.8 \text{ in.} ◀$$

Chapter 13, Solution 63.



$$\theta = 30^\circ = \frac{\pi}{6}\text{ rad}$$

$$R = 0.3\text{ m}$$

(*a*) Maximum height

Above *B* is reached when the velocity at *E* is zero

$$T_C = 0$$

$$T_E = 0$$

$$V = V_e + V_g$$

Point C

$$\Delta L_{BC} = (0.3\text{ m})\left(\frac{\pi}{6}\text{ rad}\right)$$

$$\Delta L_{BC} = \frac{\pi}{20}\text{ m}$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2 = \frac{1}{2}(40\text{ N/m})\left(\frac{\pi}{20}\text{ m}\right)^2 = 0.4935\text{ J}$$

$$(V_C)_g = WR(1-\cos\theta) = (0.2\text{ kg}\times 9.81\text{ m/s}^2)(0.3\text{ m})(1-\cos 30^\circ)$$

$$(V_C)_g = 0.07886\text{ J}$$

$$(V_E)_e = 0 \qquad \text{(spring is unattached)}$$

$$(V_E)_g = WH = (0.2\times 9.81)(H) = 1.962H\text{ (J)}$$

$$T_C + V_C = T_E + V_E$$

$$0 + 0.4935 + 0.07886 = 0 + 0 + 1.962H$$

$$H = 0.292\text{ m} ◀$$

continued

(*b*) The maximum velocity is at *B* where the potential energy is zero, $v_B = v_{\max}$

$$T_C = 0 \qquad V_C = 0.4935 + 0.07886 = 0.5724 \text{ J}$$

$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2}(0.2 \text{ kg}) v_{\max}^2$$

$$T_B = 0.1 v_{\max}^2$$

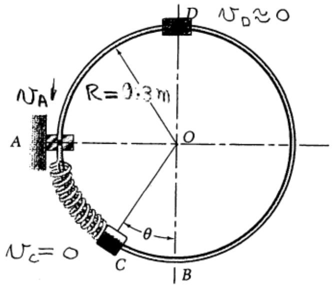
$$V_B = 0$$

$$T_C + V_C = T_B + V_B \qquad 0 + 0.5724 = (0.1) v_{\max}^2$$

$$v_{\max}^2 = 5.72 \text{ m}^2/\text{s}^2$$

$$v_{\max} = 2.39 \text{ m/s} \blacktriangleleft$$

Chapter 13, Solution 64.



$R = 0.3\text{ m}$

$W = (0.2\text{ kg}) \times (9.81\text{ m/s}^2)$

$= 1.962\text{ N}$

(*a*) *Smallest angle* θ occurs when the *velocity at D is close to zero*

$$v_C = 0 \qquad v_D = 0$$

$$T_C = 0 \qquad T_D = 0$$

$$V = V_e + V_g$$

Point C

$$\Delta L_{BC} = (0.3\text{ m})\theta = 0.3\theta\text{ m}$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2$$

$$(V_C)_e = 1.8\theta^2$$

$$(V_C)_g = WR(1-\cos\theta)$$

$$(V_C)_g = (1.962\text{ N})(0.3\text{ m})(1-\cos\theta)$$

$$V_C = (V_C)_e + (V_C)_g = 1.8\theta^2 + 0.5886(1-\cos\theta)$$

Point D

$$(V_D)_e = 0 \qquad \text{(spring is unattached)}$$

$$(V_D)_g = W(2R) = (2)(1.962\text{ N})(0.3\text{ m}) = 1.1772\text{ J}$$

$$T_C + V_C = T_D + V_D; \qquad 0 + 1.8\theta^2 + 0.5586(1-\cos\theta) = 1.1772\text{ J}$$

$$(1.8)\theta^2 - (0.5886)\cos\theta = 0.5886$$

By trial $\theta = 0.7522\text{ rad}$

$\theta = 43.1°$ ◀

continued

(*b*) *Velocity at A*
Point D

$$V_D = 0 \qquad T_D = 0 \qquad V_D = 1.1772\text{ J}\ \big(\text{see Part }(a)\big)$$

Point A

$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.2\text{ kg})v_A^2$$

$$T_A = 0.1v_A^2$$

$$V_A = (V_A)_g = W(R) = (1.962\text{ N})(0.3\text{ m}) = 0.5886\text{ J}$$

$$T_A + V_A = T_D + V_D$$

$$0.1v_A^2 + 0.5886 = 0 + 1.1772$$

$$v_A^2 = 5.886\text{ m}^2/\text{s}^2$$

$$v_A = 2.43\text{ m/s} \blacktriangleleft$$

Chapter 13, Solution 65.

Conservation of energy

Position (1) is at the top of the incline; position (2) is when the spring has maximum deformation

$$k = 1500 \text{ lb/ft}$$

Where $T_1 + V_1 = T_2 + V_2$

At (1) $$T_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{200}{32.2}\right)(8)^2 = 198.76 \text{ ft}\cdot\text{lb}$$

$$V_1 = V_{g1} + V_{e1} = mgz_1 = \frac{1}{2}k\,x_1^2 \left(\text{datum at point 2}\right)$$

$$= 200(25 - x)\sin 20° + \frac{1}{2}(1500)(0.5)^2$$

$x =$ Deformation of the spring

$$V_1 = 1710.1 + 68.404x + 187.5$$

At (2) $T_2 = 0;$ $$V_2 = \cancelto{0}{V_{g2}} + V_{e2} = \frac{1}{2}k\,x_2^2 = \frac{1}{2}(1500)(0.5 + x)^2$$

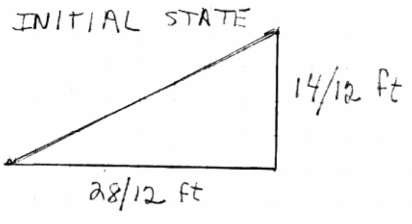
Substituting into (1) $$198.78 + 1710.1 + 68.404x + 187.5 = 750(0.5 + x)^2$$

Solve $$750x^2 + 681.596x - 1908.9 = 0$$

$$x = -2.11 \quad \text{or} \quad +1.2044 \text{ ft}$$

$$x = 1.204 \text{ ft} \blacktriangleleft$$
$$= 14.45 \text{ in.} \blacktriangleleft$$

Chapter 13, Solution 66.



$$\text{Spring length} = \sqrt{14^2 + 28^2}$$

$$= 31.305 \text{ in.}$$

$$\text{Stretch} = \frac{31.305 \text{ in.} - 14 \text{ in.}}{12 \text{ in./ft}}$$

$$= 1.44208 \text{ ft}$$

$$T_0 + V_0 = 0 + 0 + \frac{1}{2}(48 \text{ lb/ft})(1.44208 \text{ ft})^2 = 49.910 \text{ lb}\cdot\text{ft}$$

(*a*) At *A*:

$$49.910 = \frac{1}{2}\left(\frac{10 \text{ lb}}{32.2 \text{ ft/s}^2}\right)v_A^2 + \frac{1}{2}(48 \text{ lb/ft})\left(\frac{14\sqrt{2} - 14}{12}\right)^2$$

$$v_A = 16.89 \text{ ft/s} \blacktriangleleft$$

(*b*) At *B*:

$$49.910 = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2 + \frac{1}{2}(48)\left(\frac{14}{12}\right)^2 - 10\left(\frac{14}{12}\right)$$

$$v_B = 13.64 \text{ ft/s} \blacktriangleleft$$

Chapter 13, Solution 67.

$$T_1 = 0,\ V_{1e} = V_{1g} = 0 \qquad \text{Constraint: } y_B(\downarrow) = 2x_A(\rightarrow)$$

$$T_2 = \frac{1}{2}M_A v_A^2 + \frac{1}{2}M_B v_B^2$$

$$= \frac{1}{2}(4\text{ kg})\left(\frac{v_B}{2}\right)^2 + \frac{1}{2}(1.5\text{ kg})v_B^2 = 1.25v_B^2$$

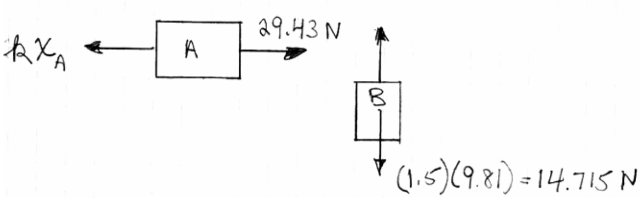
(*a*) $y_B = 0.15\text{ m},\ x_A = 0.075\text{ m},\ V_{2e} = \frac{1}{2}(300\text{ N/m})(0.075\text{ m})^2 = 0.84375\text{ N}\cdot\text{m}$

$$V_{2g} = -(1.5)(9.81)(0.15\text{ m}) = -2.2073\text{ J}$$

$$T_1 + V_1 = T_2 + V_2;\ 0 = 1.25\,v_B^2 + 0.84375 - 2.20725$$

$v_B = 1.044$ m/s ◀

(*b*) Maximum velocity when acceleration = 0



$$\sum F_x = ma_x;\quad k\,x_A - 2T = 0;\quad T = \text{Cord tensile force}$$

$$x_A = \frac{2T}{k} = \frac{2(14.715)}{300};\ x_A = 0.0981\text{ m};\ x_B = 2x_A = 0.1962\text{ m}$$

$$V_{2g} = (-14.715\text{ N})(0.1962\text{ m}) = -2.8871$$

$$V_{2e} = \frac{1}{2}(300\text{ N/m})(0.0981\text{ m})^2 = 1.4435$$

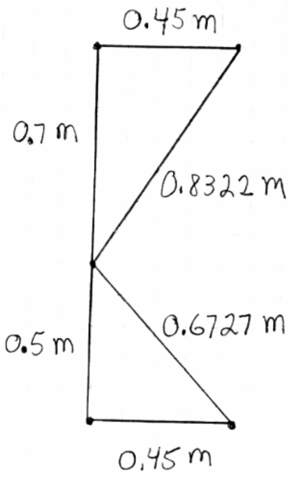
$$T_1 + V_1 = T_2 + V_2;\ 0 = 1.25v_B^2 - 1.44354$$

$v_B = 1.075$ m/s ◀

(*c*) $T_2 = 0;\ V_2 = 0 = \frac{1}{2}(300)\left(\frac{y_B}{2}\right)^2 - 14.715\,y_B$

$y_B = 0.392\text{ m} = 392\text{ mm}$ ◀

Chapter 13, Solution 68.



(*a*) Calculate spring lengths after deflection.

Original spring length = 0.75 m, collar moved 100 mm = 0.1 m

$$T_1 = V_1 = 0,\ \ T_2 = \frac{1}{2}(4)v^2 = 2v^2$$

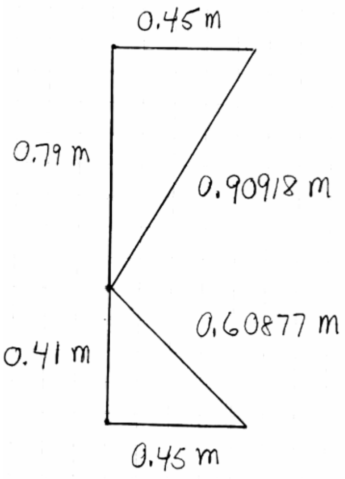
$$V_{2g} = -(4\text{ kg})\left(9.81\text{ m/s}^2\right)(0.1\text{ m}) = -3.924\text{ J}$$

$$V_{2e} = \frac{1}{2}(300)\left[(0.8322-0.75)^2 + (0.6727-0.75)^2\right]$$

$$= 1.9098\text{ J}$$

$$T_1 + V_1 = T_2 + V_2:\ \ 0 = 2v^2 - 3.924 + 1.9098$$

$v = 1.0035$ m/s $\qquad$ $v = 1.004$ m/s ◀



(*b*) Calculate spring lengths after deflection, collar moved 190 mm = 0.19 m

$$V_{2g} = -(4\text{ kg})\left(9.81\text{ m/s}^2\right)(0.19\text{ m}) = -7.4556\text{ J}$$

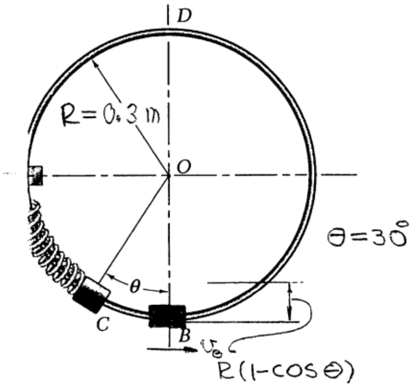
$$V_{2e} = \frac{1}{2}(300)\left[(0.90918-0.75)^2 + (0.60877-0.75)^2\right]$$

$$= 6.7927\text{ J}$$

$$T_1 + V_1 = T_2 + V_2:\ \ 0 = 2v^2 - 7.4556 + 6.7927$$

$v = 0.576$ m/s $\qquad$ $v = 0.576$ m/s ◀

Chapter 13, Solution 69.



(*a*)

$$v_C = 0, \qquad T_C = 0$$

$$T_B = \frac{1}{2}mv_B^2$$

$$T_B = \frac{1}{2}(0.2\text{ kg})v_B^2$$

$$T_B = 0.1v_B^2 \qquad V_C = (V_C)_e + (V_C)_g$$

$$\text{arc } BC = \Delta L_{BC} = R\theta$$

$$\Delta L_{BC} = (0.3\text{ m})(30°)\frac{(\pi)}{180°}$$

$$\Delta L_{BC} = 0.15708\text{ m}$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2 = \frac{1}{2}(40\text{ N/m})(0.15708\text{ m})^2 = 0.49348\text{ J}$$

$$(V_C)_g = WR(1-\cos\theta) = (0.2\text{ kg})\left(9.81\text{ m/s}^2\right)(0.3\text{ m})(1-\cos 30°)$$

$$(V_C)_g = 0.078857\text{ J}$$

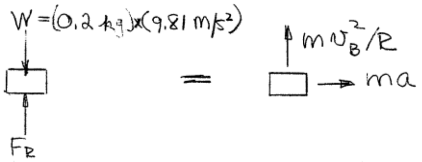
$$V_C = (V_C)_e + (V_C)_g = 0.49348\text{ J} + 0.078857\text{ J} = 0.57234\text{ J}$$

$$V_B = (V_B)_e + (V_B)_g = 0 + 0 = 0$$

$$T_C + V_C = T_B + V_B; \qquad 0 + 0.57234 = 0.1v_B^2$$

$$v_B^2 = 5.7234\text{ m}^2/\text{s}^2 \qquad v_B = 2.39\text{ m/s} ◀$$

(*b*)

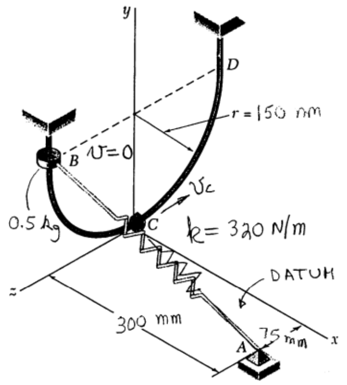


$$+\uparrow \Sigma F = F_R - W = \frac{mv_B^2}{R}$$

$$F_R = 1.962\text{ N} + (0.2\text{ kg})\frac{\left(5.7234\text{ m}^2/\text{s}^2\right)}{(0.3\text{ m})}$$

$$F_R = 1.962\text{ N} + 3.8156\text{ N} = 5.7776\text{ N} \qquad F_R = 5.78\text{ N}\uparrow ◀$$

Chapter 13, Solution 70.



(*a*) Speed at *C*

$$L_{AB} = \sqrt{(300)^2 + (150)^2 + (75)^2} = 343.69318 \text{ mm}$$

$$k = 320 \text{ N/m}$$

At B $\quad v_B = 0 \qquad T_B = 0$

$$V_B = (V_B)_e + (V_B)_g$$

$$\Delta L_{AB} = 343.69318 \text{ mm} - 200 \text{ mm}$$

$$\Delta L_{AB} = 143.69318 \text{ mm} = 0.14369318 \text{ m}$$

$$(V_B)_e = \frac{1}{2}k(\Delta L_{AB})^2 = \frac{1}{2}(320 \text{ N/m})(0.1436932 \text{ m})^2$$

$$(V_B)_e = 3.303637 \text{ J}$$

$$(V_B)_g = Wr = (0.5 \text{ kg})\left(9.81 \text{ m/s}^2\right)(0.15 \text{ m}) = 0.73575 \text{ J}$$

$$V_B = (V_B)_e + (V_B)_g = 3.303637 \text{ J} + 0.73575 \text{ J} = 4.03939 \text{ J}$$

At C $\quad T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}(0.5 \text{ kg})\left(v_C^2\right)$

$$T_C = 0.25v_C^2$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{AC})^2$$

$$\Delta L_{AC} = 309.23 \text{ mm} - 200 \text{ mm} = 109.23 \text{ mm} = 0.10923 \text{ m}$$

$$(V_C)_e = \frac{1}{2}(320 \text{ N/m})(0.10923 \text{ m})^2 = 1.90909 \text{ J}$$

$$T_B + V_B = T_C + V_C$$

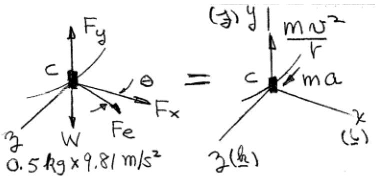
$$0 + 4.0394 = 0.25v_C^2 + 1.90909$$

$$v_C^2 = \frac{4.0394 - 1.90909}{0.25} = 8.5212\ \text{m}^2/\text{s}^2 \qquad v_C = 2.92\ \text{m/s} \blacktriangleleft$$

(*b*) Force of rod on collar *AC*

$$F_z = 0 \quad \text{(no friction)}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$



$$\theta = \tan^{-1}\frac{75}{300} = 14.04°$$

$$\mathbf{F_e} = (k\Delta L_{AC})(\cos\theta\mathbf{i} + \sin\theta\mathbf{k})$$

$$\mathbf{F_e} = (320)(0.10923)(\cos 14.04°\mathbf{i} + \sin 14.04°\mathbf{k})$$

$$\mathbf{F_e} = 33.909\mathbf{i} + 8.4797\mathbf{k}\ \text{(N)}$$

$$\Sigma\mathbf{F} = (F_x + 33.909)\mathbf{i} + (F_y - 4.905)\mathbf{j} + 8.4797\mathbf{k} = \frac{mv^2}{r}\mathbf{j} + mg\mathbf{k}$$

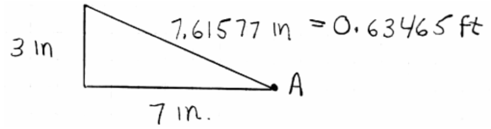
$$F_x + 33.909\ \text{N} = 0 \qquad F_y = 4.905\ \text{N} + (0.5)\frac{(8.5212\ \text{m}^2/\text{s}^2)}{0.15\ \text{m}}$$

$$F_x = -33.909\ \text{N}$$

$$F_y = 33.309\ \text{N}$$

$$\mathbf{F} = -33.9\ \text{N}\,\mathbf{i} + 33.3\ \text{N}\,\mathbf{j} \blacktriangleleft$$

Chapter 13, Solution 71.



Datum at point *C*. $m = \left(\dfrac{2.5\text{ lb}}{32.2\text{ ft/s}^2}\right) = 0.07764\ \dfrac{\text{lb}\cdot\text{s}^2}{\text{ft}}$

$$T_C = 0,\ \left(V_C\right)_g = 0,\ \left(V_C\right)_e = 0,\ T_A = \frac{1}{2}\left(\frac{2.5}{32.2}\right)v_A^2$$

$$\left(V_A\right)_g = (-2.5\text{ lb})(7/12\text{ ft}) = -1.4583$$

(*a*) $$\left(V_A\right)_C = \frac{1}{2}(20\text{ lb/ft})(0.63465\text{ ft} - 0.3333\text{ ft})^2 = 0.90812$$

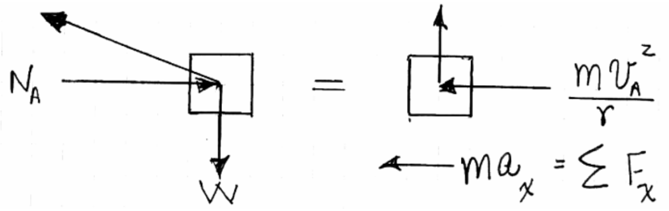
From conservation of energy:

$$0 = \frac{1}{2}(0.07764)v_A^2 - 1.4583 + 0.90812$$

$$V_A = 3.7646\text{ ft/s}$$ $v_A = 3.76\text{ ft/s}$ ◀

At point *A*,

$$F_S = k\,\Delta L_{CA} = (20\text{ lb/ft})(0.63465\text{ ft} - 0.3333\text{ ft}) = 6.0263\text{ lb}$$



$$\frac{mv_A^2}{r} = \frac{\left(2.5\text{ lb}/32.2\text{ ft/s}^2\right)(3.76551\text{ ft/s})^2}{(7/12)\text{ft}} = 1.8872\text{ lb}$$

$$\xleftarrow{+}\ \sum F = \frac{m v_A^2}{r}; \quad \frac{7}{7.61577}\left(6.0263\right) - N_A = 1.8872$$

$N_A = 3.65$ lb → ◀

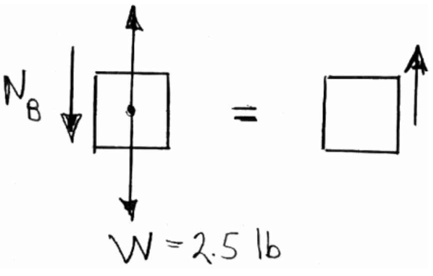
(*b*) Datum at *C*.

$$\left(V_B\right)_g = -\left(2.5\text{ lb}\right)\left(14/12\text{ ft}\right) = -2.9167\text{ ft}\cdot\text{lb}$$

$$\left(V_B\right)_e = \frac{1}{2}k\left(\Delta L_{CB}\right)^2 = \frac{1}{2}\left(20\text{ lb/ft}\right)\left(0.5\text{ ft}\right)^2 = 2.5\text{ ft}\cdot\text{lb}$$

From Conservation of energy:

$$0 + \frac{1}{2}\left(\frac{2.5}{32.2}\right)v_B^2 - 2.9167 + 2.5$$



$v_B = 3.2762$ ft/s $\qquad$ $v_B = 3.28$ ft/s ◀

$$F_S = k\left(\Delta L_{CB}\right) = \left(20\right)\left(0.5\right) = 10\text{ lb}$$

$$\left(\frac{2.5}{32.2}\right)\frac{v_B^2}{\left(7/12\right)} = 1.4286\text{ lb}$$

$$+\!\uparrow \sum F_y = -N_B + 10 - 2.5 = 1.4286$$

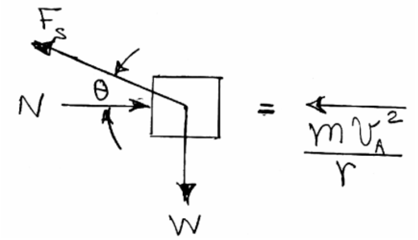
$N_B = 6.07$ lb ↓ ◀

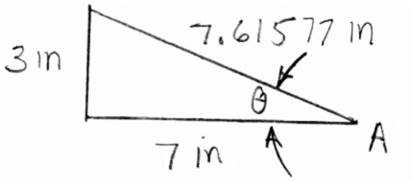
Chapter 13, Solution 72.

(*a*) For maximum velocity,

$$\Delta L = 7.61577 - 4 = 3.61577 \text{ in.} = 0.30131 \text{ ft}$$

$$a_t = \ddot{s} = 0 \qquad \sin\theta = (3/7.61577)$$





$$W = 2.5 \text{ lb}$$

$$+\uparrow \sum F_y = 0 = 0.30131\,k(3/7.61577) - 2.5 = 0$$

$$k = 21.063 \text{ lb/ft} \qquad k = 21.1 \text{ lb/ft} \blacktriangleleft$$

(*b*) Put datum at *C*

$$T_C = (V_C)_g = (V_C)_e = 0, \qquad T_A = \frac{1}{2}\left(\frac{2.5}{32.2}\right)v_A^2$$

$$(V_A)_g = -2.5(7/12) = -1.4583$$

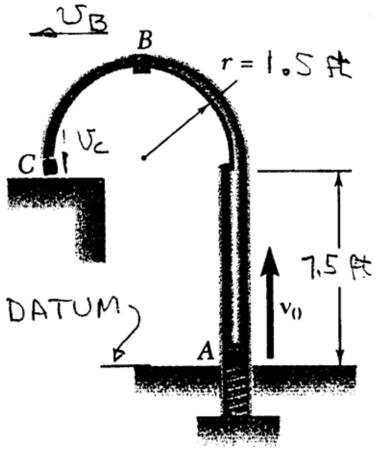
$$(V_A)_e = \frac{1}{2}(21.063)(0.30131)^2 = 0.9561$$

Conservation of energy: $0 = \dfrac{1}{2}\left(\dfrac{2.5}{32.2}\right)v_A^2 - 1.4583 + 0.9561$

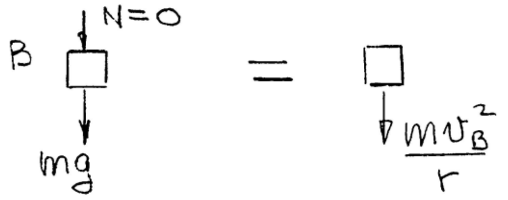
$$v_A = 3.597 \qquad v_A = 3.60 \text{ ft/s} \blacktriangleleft$$

Chapter 13, Solution 73.

Loop 1



(*a*) The smallest velocity at *B* will occur when the force exerted by the tube on the package is zero.



$$+\downarrow \Sigma F = 0 + mg = \frac{mv_B^2}{r}$$

$$v_B^2 = rg = 1.5 \text{ ft}\left(32.2 \text{ ft/s}^2\right)$$

$$v_B^2 = 48.30$$

At *A* $\quad T_A = \frac{1}{2} m\, v_0^2$

$$V_A = 0 \quad \left(8 \text{ oz} = 0.5 \text{ lb} \Rightarrow = \frac{0.5}{32.2} = 0.01553\right)$$

At *B* $\quad T_B = \frac{1}{2} mv_B^2 = \frac{1}{2} m(48.30) = 24.15 \text{ m}$

$$V_B = mg(7.5 + 1.5) = 9mg = 9(0.5) = 4.5 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B: \quad \frac{1}{2}(0.01553)v_0^2 = 24.15(0.01553) + 4.5$$

$$v_0^2 = 627.82 \qquad v_0 = 25.056 \qquad v_0 = 25.1 \text{ ft/s} \blacktriangleleft$$

At *C*

$$T_C = \frac{1}{2} mv_C^2 = 0.007765 v_C^2 \qquad v_C = 7.5mg = 7.5(0.5) = 3.75$$

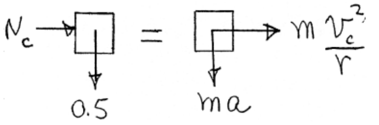
$$T_A + V_A = T_C + V_C: \quad 0.007765 v_0^2 = 0.007765 v_C^2 + 3.75$$

$$0.007765(25.056)^2 - 3.75 = 0.007765 v_C^2$$

$$v_C^2 = 144.87$$

continued

(*b*)

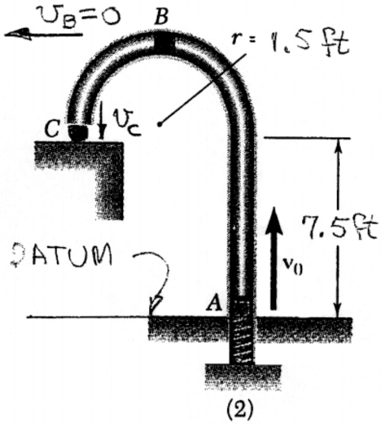


$$\xrightarrow{+} \Sigma F = ma_n: \quad N = 0.01553\frac{(144.87)}{1.5}$$

$$N = 1.49999$$

{Package in tube} $N_C = 1.500$ lb ⟵ ◀

Loop 2



(*a*) At *B*, tube supports the package so,

$$v_B \approx 0$$

$$v_B = 0, \; T_B = 0 \qquad V_B = mg(7.5 + 1.5)$$

$$= 4.5 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B$$

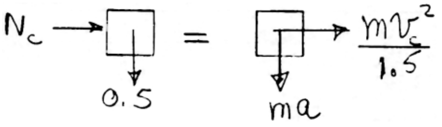
$$\frac{1}{2}(0.01553)v_A^2 = 4.5 \Rightarrow v_A = 24.073$$

$v_A = 24.1$ ft/s ◀

(*b*) At *C* $\qquad T_C = 0.007765v_C^2, \; V_C = 7.5mg = 3.75$

$$T_A + V_A = T_C + V_C: \quad 0.007765(24.073)^2 = 0.007765v_C^2 + 3.75$$

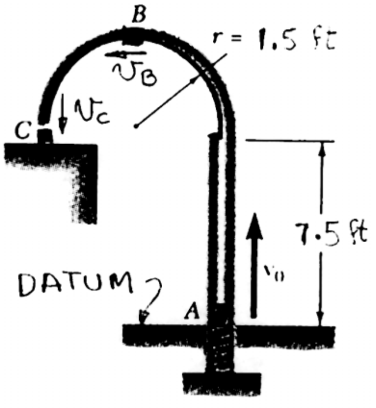
$$v_C^2 = 96.573$$



$$N_C = 0.01553\left(\frac{96.573}{1.5}\right) = 0.99985$$

{Package on tube} $N_C = 1.000$ lb ◀

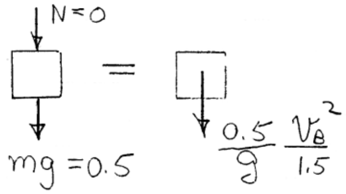
Chapter 13, Solution 74.



(1)

(*a*) *Loop 1*

From 13.75, at *B*



$$v_B^2 = gr = 48.3 \text{ ft}^2/\text{s}^2 \Rightarrow v_B = 6.9498 \text{ ft/s}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.01553)(48.3) = 0.37505$$

$$V_B = mg(7.5 + 1.5) = (0.5)(9) = 4.5 \text{ lb}\cdot\text{ft}$$

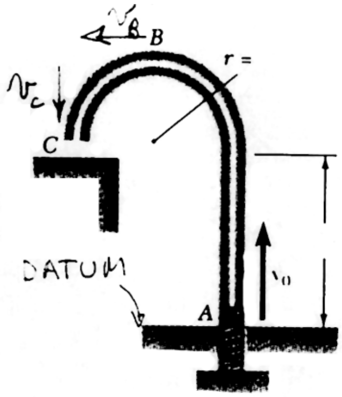
$$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}(0.01553)v_C^2 = 0.007765v_C^2$$

$$V_C = 7.5(0.5) = 3.75 \text{ lb}\cdot\text{ft}$$

$$T_B + V_B = T_C + V_C: \quad 0.37505 + 4.5 = 0.007765v_C^2 + 3.75$$

$$v_C^2 = 144.887 \Rightarrow v_C = 12.039 \text{ ft/s}$$

12.04 ft/s > 10 ft/s ⇒ Loop (1) does not work ◀



(2)

(*b*) *Loop 2 at A* $T_A = \frac{1}{2}mv_0^2 = 0.007765v_0^2$

$V_A = 0$

At C assume $v_C = 10$ ft/s

$$T_C = \frac{1}{2}mv_C^2 = 0.007765(10)^2 = 0.7765$$

$$v_C = 7.5(0.5) = 3.75$$

$$T_A + V_A = T_C + V_C: \quad 0.007765v_0^2 = 0.7765 + 3.75$$

$v_0 = 24.144$ $v_0 = 24.1$ ft/s ◀

Chapter 13, Solution 75.

Use conservation of energy from the point of release (*A*) and the top of the circle.

$$T_1 + V_1 = T_2 + V_2 \qquad \text{(1) (datum at lowest point)}$$

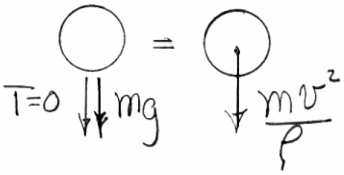
where

$$T_1 = 0; \quad V_1 = mg\ell$$

At 2 $\qquad T_2 = \frac{1}{2}mv^2; \; V_2 = mgz = mg(2)(\ell - a)$

Substituting into (1) $\qquad 0 + mg\ell = \frac{1}{2}mv^2 + 2mg(\ell - a) \qquad (2)$

We need another equation – use Newton's 2nd law at the top. $\left(\text{Tension, } T_0 = 0 \text{ at top}\right)$



$$+\!\downarrow \sum F_n = ma_n \Rightarrow \cancel{m}g = \frac{\cancel{m}v^2}{\rho}$$

$$v^2 = g\rho = g(\ell - a)$$

Substituting into (2)

$$mg\ell = \frac{1}{2}mg(\ell - a) + 2mg(\ell - a)$$

$$2\ell = \ell - a + 4\ell - 4a$$

$$5a = 3\ell$$

$$a = \frac{3}{5}\ell \blacktriangleleft$$

Chapter 13, Solution 76.

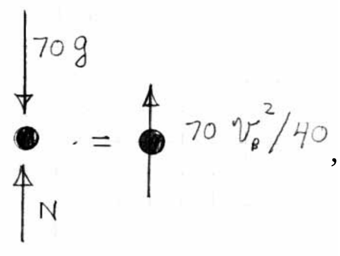
$$T_A = V_A = 0,\quad T_B = \frac{1}{2}(70)v_B^2,\quad V_B = -70\ \text{kg}\left(9.81\ \text{m/s}^2\right)(40\ \text{m})(1-0.7071)$$

$$T_C = \frac{1}{2}(70)\,v_C^2,\quad V_C = -2V_B$$

Conservation of energy: $T_B + V_B = 0,\quad v_B = 15.161$ m/s

$T_C + V_C = 0,\quad v_C = 21.441$ m/s

(*a*)

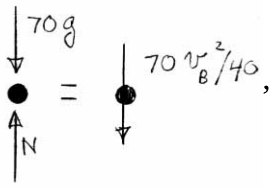


$$N = \left(70\ v_B^2/40\right) + 70\ g$$

$$N = 402.26 + 686.7$$

$N_{B1} = 1089$ N ↑◀

(*b*)

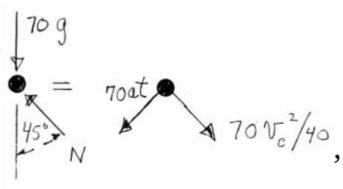


$$N = \left(-70\ v_B^2/40\right) + 70\ g$$

$$N = -402.26 + 686.7$$

$N_{B2} = 284$ N ↑◀

(*c*)



$$N - 70\ \text{kg}(0.7071) = -70\ v_C^2/40$$

$$N_{C1} = -804.5 + 485.6 < 0$$

Skier airborne? Yes◀

Chapter 13, Solution 77.

For a conservative force, Equation (13.22) must be satisfied

$$F_x = -\frac{\partial V}{\partial x} \qquad F_y = -\frac{\partial V}{\partial y} \qquad F_z = -\frac{\partial V}{\partial z}$$

We now write $$\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x \partial y} \qquad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y \partial x}$$

Since $$\frac{\partial^2 V}{\partial x \partial y} = \frac{\partial^2 V}{\partial y \partial x}: \quad \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \blacktriangleleft$$

We obtain in a similar way

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \qquad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \blacktriangleleft$$

Chapter 13, Solution 78.

(*a*) $$F_x = \frac{yz}{xyz} \qquad F_y = \frac{zx}{xyz}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(\frac{1}{x}\right)}{\partial y} = 0 \qquad \frac{\partial F_y}{\partial x} = \frac{\partial\left(\frac{1}{y}\right)}{\partial x} = 0$$

Thus $$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The other two equations derived in Problem 13.80 are checked in a similar way.

(*b*) Recall that $$F_x = -\frac{\partial v}{\partial x}, \qquad F_y = -\frac{\partial v}{\partial y}, \qquad F_z = -\frac{\partial v}{\partial z}$$

$$F_x = \frac{1}{x} = -\frac{\partial v}{\partial x} \qquad V = -\ln x + f(y,z) \tag{1}$$

$$F_y = \frac{1}{y} = -\frac{\partial v}{\partial y} \qquad V = -\ln y + g(z,x) \tag{2}$$

$$F_z = \frac{1}{z} = -\frac{\partial v}{\partial z} \qquad V = -\ln z + h(x,y) \tag{3}$$

Equating (1) and (2)

$$-\ln x + f(y,z) = -\ln y + g(z,x)$$

Thus $$f(y,z) = -\ln y + k(z) \tag{4}$$

$$g(z,x) = -\ln x + k(z) \tag{5}$$

Equating (2) and (3)

$$-\ln z + h(x,y) = -\ln y + g(z,x)$$

$$g(z,x) = -\ln z + l(x)$$

From (5)

$$g(z,x) = -\ln x + k(z)$$

Thus

$$k(z) = -\ln z$$

$$l(x) = -\ln x$$

continued

From (4)

$$f(y,z) = -\ln y - \ln z$$

Substitute for $f(y,z)$ in (1)

$$V = -\ln x - \ln y - \ln z$$

$$V = -\ln xyz \blacktriangleleft$$

Chapter 13, Solution 79.

(*a*) $$F_x = \frac{x}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}} \qquad F_y = \frac{y}{\left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}}$$

$$\frac{\partial F_x}{\partial y} = \frac{x\left(-\frac{3}{2}\right)(2y)}{\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} \qquad \frac{\partial F_y}{\partial x} = \frac{y\left(-\frac{3}{2}\right)2y}{\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}}$$

Thus $$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The other two equations derived in Problem 13.79 are checked in a similar fashion

(*b*) Recalling that $$F_x = -\frac{\partial V}{\partial x},\ F_y = -\frac{\partial V}{\partial y},\ F_z = -\frac{\partial V}{\partial z}$$

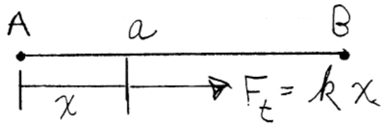
$$F_x = -\frac{\partial V}{\partial x} \qquad V = -\int \frac{x}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}} dx$$

$$V = \left(x^2 + y^2 + z^2\right)^{-\frac{1}{2}} + f(y,z)$$

Similarly integrating $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$ shows that the unknown function $f(x,y)$ is a constant.

$$V = \frac{1}{\left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}} \blacktriangleleft$$

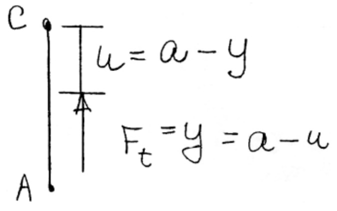
Chapter 13, Solution 80.



(*a*)

$$U_{AB} = \int_0^a kx\,dx = k\frac{a^2}{2}$$

$F_x = F_y$, $\therefore$ F is normal *to BC*, $U_{BC} = 0$



$$U_{CA} = \int_0^a -(a-u)\,du = \frac{-a^2}{2}$$

$$U_{ABCA} = (k-1)\frac{a^2}{2}, \text{ not conservative} \blacktriangleleft$$

(*b*) From Problem 13.77,

$$\frac{\partial F_x}{\partial y} = 1 = \frac{\partial F_y}{\partial x}$$

Conservative, $U_{ABCA} = 0 \blacktriangleleft$

Chapter 13, Solution 81.

(*a*)
$$U_{1\to 2} = -\int_{x_1}^{x_2} F dx = -\int_{x_1}^{x_2} \left(k_1 x + k_2 x^3\right) dx$$

$$= \frac{k_1}{2}\left(x_2^2 - x_1^2\right) - \frac{k_2}{4}\left(x_2^4 - x_1^4\right)$$

$$U_{1\to 2} = V_{e1} - V_{e2}$$

$$V_e = \frac{1}{2}k_1 x^2 + \frac{1}{4}k_2 x^4 \blacktriangleleft$$

(*b*) Conservation of energy:
$$T_1 = 0,\ T_2 = \frac{1}{2}mv^2$$

$$V_{1e} = \frac{1}{2}k_1 x_0^2 + \frac{1}{4}k_2 x_0^4,\ V_{2e} = 0$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_1 x_0^2 + \frac{1}{4}k_2 x_0^4$$

$$v = \sqrt{\left(\frac{k_1}{m}\right)x_0^2 + \left(\frac{k_2}{2m}\right)x_0^4} \blacktriangleleft$$

Chapter 13, Solution 82.

(*a*)

$$U_{1\to 2} = -\int_{x_1}^{x_2} F dx = -\int_{x_1}^{x_2} \left(k_1 x + k_2 x^3\right) dx$$

$$= -\frac{k_1}{2}\left(x_2^2 - x_1^2\right) + \frac{k_2}{4}\left(x_2^4 - x_1^4\right)$$

$U_{1\to 2} = V_{e1} - V_{e2}$:

$$V_e = \frac{1}{2}k_1 x^2 - \frac{1}{4}k_2 x^4 \blacktriangleleft$$

(*b*) Conservation of energy: $T_1 = 0,\ T_2 = \frac{1}{2}mv^2$

$$V_{1e} = \frac{1}{2}k_1 x_0^2 - \frac{1}{4}k_2 x_0^4,\ V_{2e} = 0$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_1 x_0^2 - \frac{1}{4}k_2 x_0^4$$

$$v = \sqrt{\left(\frac{k_1}{m}\right)x_0^2 + \left(\frac{k_2}{2m}\right)x_0^4} \blacktriangleleft$$

$$\left(\text{Requires } x_0 < \sqrt{\frac{2k_1}{k_2}}\right)$$

Chapter 13, Solution 83.

Circular orbit velocity

$$R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$$

$$r_{Ao} = 930 \text{ mi} = 4.9104 \times 10^6 \text{ ft}$$

$$\frac{v_C^2}{r} = \frac{GM}{r^2}, \quad GM = gR^2$$

$$v_C^2 = \frac{GM}{r} = \frac{gR^2}{r} = \frac{(32.2 \text{ ft/s})\left(20.9088 \times 10^6 \text{ ft}\right)^2}{\left(20.9088 \times 10^6 \text{ ft} + 4.9104 \times 10^6 \text{ ft}\right)}$$

$$v_C^2 = 545.22 \times 10^6 \text{ ft}^2/\text{s}^2$$

$$v_C = 23350 \text{ ft/s}$$

Velocity reduced to 60% of $v_C = 14010$ ft/s

Conservation of energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\cancel{m}v_A^2 - \frac{GM\cancel{m}}{r_A} = \frac{1}{2}\cancel{m}v_B^2 - \frac{GM\cancel{m}}{r_B}$$

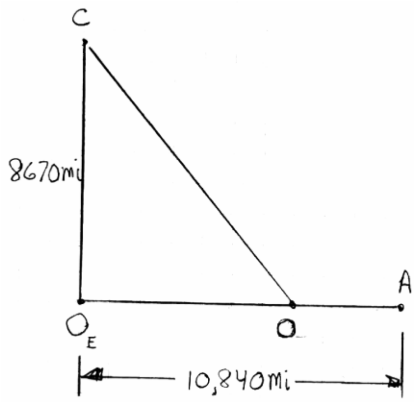
$$\frac{1}{2}(14010)^2 - \frac{32.2\left(20.9088 \times 10^6\right)^2}{\left(25.819 \times 10^6\right)} = \frac{v_B^2}{2} - \frac{32.2\left(20.9088 \times 10^6\right)^2}{\left(20.9088 \times 10^6\right)}$$

$$v_B = 21269 \text{ ft/s}$$

$v_B = 4.03$ mi/s ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 84.



Distance $\quad OA = 3960 \text{ mi} + 376 \text{ mi} = 4336 \text{ mi} = 22.894 \times 10^6 \text{ ft}$

Distance $\quad OO_E = 10{,}840 \text{ mi} - 4336 \text{ mi} = 6504 \text{ mi} = 34.341 \times 10^6 \text{ ft}$

$$r_C^2 = (8670)^2 + (6504)^2$$

$$r_C = 10{,}838.4 \text{ mi} = 57.2268 \times 10^6 \text{ ft}$$

$$T + V = \frac{1}{2}m(15681.6)^2 - \frac{gR^2 m}{r_C}$$

$$\text{constant} = \frac{1}{m}(T + V) = -123.032 \times 10^6 \text{ ft}^2/\text{s}^2$$

$$R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}; \; v_C = 2.97 \text{ mi/s} = 15681.6 \text{ ft/s}$$

(*a*) At point *A*,

$$\frac{T + V}{m} = \frac{1}{2}v_A^2 - \frac{gR^2}{22.894 \times 10^6 \text{ ft}}$$

$$\left(-123.032 \times 10^6\right) = \frac{1}{2}v_A^2 - \frac{\left(32.2 \text{ ft/s}^2\right)\left(20.9088 \times 10^6 \text{ ft}\right)^2}{\left(22.894 \times 10^6 \text{ ft}\right)}$$

$$v_A = 31364 \text{ ft/s} \qquad v_A = 5.94 \text{ mi/s} \blacktriangleleft$$

(*b*) At point *B*,

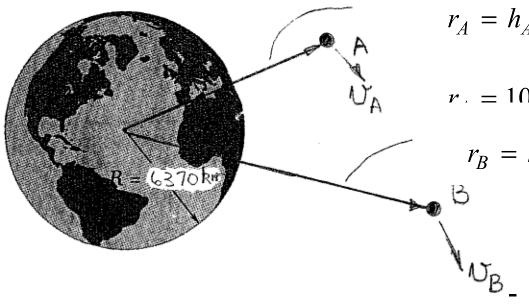
$$r_B = 2(10{,}840 \text{ mi}) - (4336 \text{ mi})$$

$$= 17344 \text{ mi} = 91.5763 \times 10^6 \text{ ft}$$

$$\left(-123.032 \times 10^6\right) = \frac{1}{2}v_B^2 - \frac{(32.2)\left(20.9088 \times 10^6\right)^2}{\left(91.5763 \times 10^6\right)}$$

$$v_B = 7834.3 \text{ ft/s} \qquad v_B = 1.484 \text{ mi/s} \blacktriangleleft$$

Chapter 13, Solution 85.



$$r_A = h_A + R = 4.3\,Mm + 6.37\,Mm$$

$$r_A = 10.67\,Mm$$

$$r_B = h_B + R = 72.7\,Mm + 6.37\,Mm$$

$$r_B = 19.07\,Mm$$

At A,

$$v_A = 32.5\,Mm/\text{h} = 9028\text{ m/s}$$

$$T_A = \frac{1}{2}m\left(9028\text{ m/s}\right)^2 = 40.752\times10^6\;m$$

$$V_A = -\frac{GMm}{r_A} = \frac{-gR^2m}{r_A}$$

$$r_A = 10.67\text{ M}m = 10.67\times10^6\text{ m}$$

$$R = 6370\text{ km} = 6.37\times10^6\text{ m}$$

$$V_A = -\frac{\left(9.81\text{ m/s}^2\right)\left(6.37\times10^6\text{ m}\right)^2}{\left(10.67\times10^6\text{ m}\right)}\text{m} = -37.306\times10^6\text{m}$$

At B

$$T_B = \frac{1}{2}mv_B^2;\; V_B = -\frac{GMm}{r_B} = \frac{-gR^2m}{r_B}$$

$$r_B = 19.07\text{ M}m = 19.07\times10^6\text{ m}$$

$$V_B = -\frac{\left(9.81\text{ m/s}^2\right)\left(6.37\times10^6\text{ m}\right)^2 m}{\left(19.07\times10^6\text{ m}\right)} = -20.874\times10^6\text{m}$$

$$T_A + V_A = T_B + V_B;\; 40.752\times10^6\text{m} - 37.306\times10^6\text{m} = \frac{1}{2}mv_B^2 - 20.874\times10^6\text{m}$$

$$v_B^2 = 2\left[40.752\times10^6 - 37.306\times10^6 + 20.874\times10^6\right]$$

$$v_B^2 = 48.64\times10^6\text{ m}^2/\text{s}^2$$

$$v_B = 6.9742\times10^3\text{ m/s} = 25.107\text{ Mm/h}$$

$$v_B = 25.1\text{ Mm/h} \blacktriangleleft$$

Chapter 13, Solution 86.

Note: $GM_{\text{moon}} = 0.0123GM_{\text{earth}}$

By Equation 12.30 $GM_{\text{moon}} = 0.0123gR_E^2$

At ∞ *distance from moon*: $r_2 = \infty$, assume $v_2 = 0$

$$E_2 = T_2 + V_2 = 0 - \frac{GM_m\, m_{\text{lem}}}{\infty} = 0 - 0 = 0$$

(*a*) On surface of moon: $R_M = 1740 \text{ km} = 1.74 \times 10^6 \text{ m}$

$v_1 = 0,\ T_1 = 0$ $R_E = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$

$$V_1 = \frac{-GM_m m_{\text{lem}}}{R_M} \qquad E_1 = T_1 + V_1 = 0 - \frac{0.0123\ gR_E^2 m_{\text{lem}}}{R_m}$$

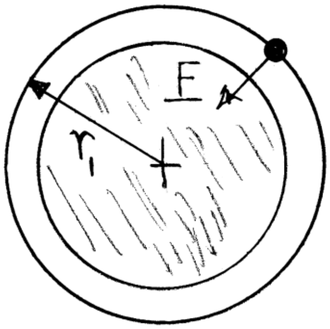
$$E_1 = -\frac{(0.0123)(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 m_{\text{lem}}}{(1.740 \times 10^6 \text{ m})}$$

Where $m_{\text{lem}} =$ mass of the lem

$$E_1 = (-2.814 \times 10^6 \text{ m}^2/\text{s}^2)\, m_{\text{lem}}$$

$$\Delta E = E_2 - E_1 = 0 + (2.814 \times 10^6 \text{ m}^2/\text{s}^2)\, m_{\text{lem}}$$

Energy per kilogram: $\dfrac{\Delta E}{m_{\text{lem}}} = 2810 \text{ kj/kg}$ ◀



(*b*) $r_1 = R_m + 80 \text{ km}$

$r_1 = (1740 \text{ km} + 80 \text{ km}) = 1820 \text{ km} = 1.82 \times 10^6 \text{ m}$

Newton's second law:

$$F = m_{\text{lem}} a_{\text{lem}}: \quad \frac{GM_m m_{\text{lem}}}{r_1^2} = \frac{m_{\text{lem}} v_1^2}{r_1}$$

$$v_1^2 = \frac{GM_m}{r_1} \qquad T_1 = \frac{1}{2} m_{\text{lem}} v_1^2 = \frac{1}{2} \frac{m_{\text{lem}} GM_m}{r_1}$$

$$V_1 = \frac{-GM_m m_{\text{lem}}}{r_1} \qquad E_1 = T_1 + V_1 = \frac{1}{2}\frac{GM_m m_{\text{lem}}}{r_1} - \frac{GM_m m_{\text{lem}}}{r_1}$$

$$E_1 = -\frac{1}{2}\frac{GM_m m_{\text{lem}}}{r_1} = -\frac{1}{2}\frac{(0.0123)\,gR_E^2 m_{\text{lem}}}{r_1}$$

$$E_1 = -\frac{1}{2}\frac{(0.0123)\left(9.81\text{ m/s}^2\right)\left(6.37\times10^3\text{ m}\right)m_{\text{lem}}}{1.82\times10^6\text{ m}}$$

$$E_1 = \left(-1.345\times10^6\text{ m}^2/\text{s}^2\right)m_{\text{lem}}$$

$$\Delta E = E_2 - E_1 = 0 + \left(1.345\times10^6\text{ m}^2/\text{s}^2\right)m_{\text{lem}}$$

Energy per kilogram: $\dfrac{\Delta E}{m_{\text{lem}}} = 1345\text{ kJ/kg}$ ◀

Chapter 13, Solution 87.

Total energy per unit weight

$$E_0 = T_A + V_A = T_p + V_p;\; E_0 = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}mv_p^2 - \frac{GMm}{r_p}$$

Unit weight $W = mg$

$$E_0 = \frac{W}{2g}v_A^2 - \frac{W}{g}\frac{GM}{r_A} = \frac{W}{2g}v_p^2 - \frac{W}{g}\frac{GM}{r_p}$$

$$\frac{E_0}{W} = \frac{v_A^2}{2g} - \frac{GM}{gr_A} = \frac{v_p^2}{2g} - \frac{GM}{gr_p} \qquad (1)$$

$$\frac{v_A^2}{2g}\left(1 - \frac{v_p^2}{v_A^2}\right) = \frac{GM}{g}\left(\frac{1}{r_A} - \frac{1}{r_p}\right)$$

$$v_A^2\left(1 - \frac{v_p^2}{v_A^2}\right) = 2GM\left(\frac{r_p - r_A}{r_A r_p}\right)$$

$$\frac{v_A}{v_p} = \frac{r_p}{r_A}\ (\text{given})$$

$$v_A^2\left(1 - \frac{r_A^2}{r_p^2}\right) = 2GM\left(\frac{r_p - r_A}{r_A r_p}\right)$$

$$v_A^2\left(\frac{r_p^2 - r_A^2}{r_p^2}\right) = 2GM\left(\frac{r_p - r_A}{r_A r_p}\right)$$

$$v_A^2 = 2GM\frac{r_p}{r_A}\left(\frac{1}{r_p + r_A}\right) \qquad (2)$$

Substitute v_A from (2) into (1)

$$\frac{E_0}{W} = \frac{1}{2g}\left[2GM\frac{r_p}{r_A}\left(\frac{1}{r_p + r_A}\right)\right] - \frac{GM}{gr_A} = \frac{GM}{g}\left[\frac{r_p}{r_A}\left(\frac{1}{r_p + r_A}\right) - \frac{1}{r_A}\right]$$

$$= \frac{GM}{r_A g}\left[\frac{r_p}{r_p + r_A} - 1\right] = \frac{GM}{r_A g}\left(\frac{r_p - \left(r_p + r_A\right)}{r_p + r_A}\right)$$

$$= \frac{-GM}{g\left(r_A + r_B\right)}$$

$$GM = gR_E^2 \Rightarrow \frac{E_0}{W} = -\frac{R_E^2}{r_A + r_p}$$

$$\frac{E_0}{W} = -\frac{\left(3960 \text{ mi} \times 5280 \text{ ft/mi}\right)^2}{\left(50{,}000 \text{ mi} \times 5280 \text{ ft/mi}\right)} = -1.65598 \times 10^6 \text{ ft} \cdot \text{lb/lb}$$

On earth: $E_E = T_E + V_E,\ V_E = 0,\ T_E = 0,\ V_E = -\dfrac{WGM}{gR_E}$

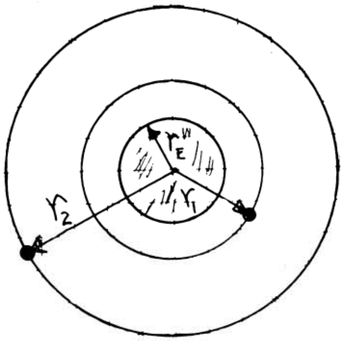
$$\frac{E_E}{W} = -\frac{GM}{gR_E} = -\frac{gR_E^2}{gR_E} = -R_E = -20.9088 \times 10^6 \text{ ft} \cdot \text{lb/lb}$$

For propulsion: $\dfrac{E_p}{W} = \dfrac{E_0}{W} - \dfrac{E_E}{W}$

$$= -1.65598 \times 10^6 - \left(-20.9088 \times 10^6\right)$$

$$\frac{E_p}{W} = 19.25 \times 10^6 \text{ ft} \cdot \text{lb/lb} \blacktriangleleft$$

Chapter 13, Solution 88.



Geosynchronous orbit

$$r_1 = 3960 + 200 = 4160 \text{ mi} = 21.965\times 10^6 \text{ ft}$$

$$r_2 = 3960 + 22{,}000 = 25{,}960 \text{ mi} = 137.07\times 10^6 \text{ ft}$$

Total energy $\quad E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r}$

M = mass of earth

m = mass of satellite

Newton's second law $\quad F = ma_n;\ \frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{GM}{r}$

$$T = \frac{1}{2}mv^2 = m\frac{GM}{2r} \qquad V = -\frac{GMm}{r}$$

$$E = T + V = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

$$GM = gR_E^2 \qquad E = -\frac{1}{2}\frac{gR_E^2 m}{r} = -\frac{1}{2}\frac{R_E^2 W}{r} \text{ where } (W = mg)$$

$$E = -\frac{1}{2}\frac{(6000)\left(20.9088\times 10^6 \text{ ft}\right)^2}{r} = -\frac{1.3115\times 10^{18}}{r} \text{ lb}\cdot\text{ft}$$

Geosynchronous orbit at $r_2 = 137.07\times 10^6$ ft

$$E_{Gs} = \frac{-1.3115\times 10^{18}}{137.07\times 10^6} = -9.5681\times 10^9 \text{ lb}\cdot\text{ft}$$

(*a*) At 200 mi, $\quad r_1 = 21.965\times 10^6$ ft

$$E_{200} = -\frac{1.3115\times 10^{18}}{21.965\times 10^6} = -5.9709\times 10^{10}$$

$$\Delta E_{300} = E_{Gs} - E_{200} = 5.0141\times 10^{10}$$

$$\Delta E_{300} = 50.1\times 10^9 \text{ ft}\cdot\text{lb} \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

(*b*) Launch from earth

At launch pad $$E_E = -\frac{GMm}{R_E} = \frac{-gR_E^2 m}{R_E} = -WR_E$$

$$E_E = -6000\left(3960 \times 5280\right) = -1.25453 \times 10^{11}$$

$$\Delta E_E = E_{Gs} - E_E = -9.5681 \times 10^9 + 125.453 \times 10^9$$

$$\Delta E_E = 115.9 \times 10^9 \text{ ft}\cdot\text{lb} \blacktriangleleft$$

Chapter 13, Solution 89.

We know

Geosynchronous orbit: $r_2 = 35780 + 6370 = 42{,}150 \text{ km}$

Orbit of shuttle: $r_1 = 6370 \text{ km} + 296 \text{ km} = 6666 \text{ km}$

Radius of Earth: $R = 6370\,\text{km}$ also $= GM = gR^2$

For any circular orbit

$$F = ma_n \Rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{GM}{r}$$

Energy

$$T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}; \qquad v = -\frac{GMm}{r}$$

$$E = T + V = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}\left(\frac{gR^2m}{r}\right)$$

For Geosynchronous orbit

$$E_2 = -\frac{1}{2}\frac{\left(9.81 \text{ m/s}^2\right)\left(6.370\times10^6 \text{ m}\right)^2\left(3600 \text{ kg}\right)}{42.150\times10^6 \text{ m}} = -16.999\times10^9 \text{ J}$$

For orbit of shuttle

$$E_1 = -\frac{1}{2}\frac{(9.81)\left(6.370\times10^6\right)^2(3600)}{6.666\times10^6} = -107.487\times10^6 \text{ J}$$

On the launching pad $v_o = 0$

$$E_o = -\frac{GMm}{R} = -mgR = -(3600)(9.81)\left(6.370\times10^6\right) = -224.963\times10^9 \text{ J}$$

(*a*) From shuttle to orbit

$$\Delta E = E_2 - E_1 = -16.999\times10^9 \text{ J} - \left(-107.487\times10^9 \text{ J}\right)$$

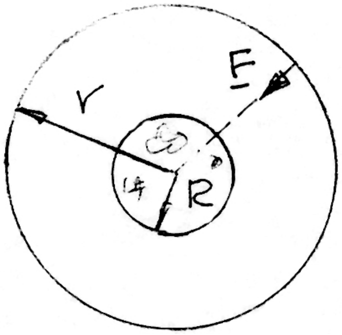
$$\Delta E = 90.5\times10^9 \text{ J} \blacktriangleleft$$

(*b*) From surface to orbit

$$\Delta E = E_2 - E_0 = -16.999\times10^9 \text{ J} - \left(-224.963\times10^9 \text{ J}\right)$$

$$\Delta E = 208\times10^9 \text{ J} \blacktriangleleft$$

Chapter 13, Solution 90.



(*a*) Potential energy $V = -\dfrac{GMm}{r} = -\dfrac{gR^2m}{r} + \text{constant}$

(*cf.* Equation 13.17)

Choosing the constant so that $V = 0$ for $r = R$:

$$V = mgR\left(1 - \frac{R}{r}\right) \blacktriangleleft$$

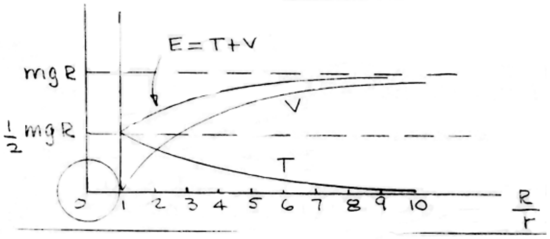
(*b*) Kinetic energy

Newton's second law $F = ma_n$: $\dfrac{GMm}{r^2} = m\dfrac{v^2}{r}$

$$v^2 = \frac{GM}{r} = \frac{gR^2}{r}$$

$$T = \frac{1}{2}mv^2 \qquad T = \frac{1}{2}\frac{mgR^2}{r} \blacktriangleleft$$

Energy



(*c*) Total energy

$$E = T + V = \frac{1}{2}m\frac{gR^2}{r} + mg\left(1 - \frac{R}{r}\right)$$

$$E = mgR\left(1 - \frac{R}{2r}\right) \blacktriangleleft$$

Chapter 13, Solution 91.

In a circular orbit, $F_N = \dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$

m = mass of Venus, M = mass of Sun

$\therefore\ v^2 = \dfrac{GM}{r}$, $T = \dfrac{1}{2}mv^2 = \dfrac{GMm}{2r}$

$$V = -\frac{GMm}{r}, \qquad T + V = -\frac{GMm}{2r}$$

(*a*)
$$M = \frac{v^2 r}{G} = \frac{\left[(78.3\times10^3)\left(\dfrac{88}{60}\right)\right]^2 (67.2\times10^6)(5280)}{34.4\times10^{-19}}$$

$M = 136.0\times10^{27}$ lb·s²/ft ◀

(*b*)
$$T + V = -\frac{GMm}{2r} = -\frac{1}{2}mv^2$$

$$T + V = -\frac{1}{2}\left(\frac{136.029\times10^{27}}{407\times10^3}\right)\left[78.3\times10^3\left(\frac{88}{60}\right)\right]^2$$

$T + V = -2.20\times10^{33}$ lb·ft ◀

Chapter 13, Solution 92.

$$V_g = -\frac{WR^2}{r} \quad \text{setting } r = R + y: \quad V_g = -\frac{WR^2}{R+y} = -\frac{WR}{1+\frac{y}{R}}$$

$$V_g = -WR\left(1+\frac{y}{R}\right)^{-1} = -WR\left[1+\frac{(-1)}{1}\frac{y}{R}+\frac{(-1)(-2)}{1\cdot 2}\left(\frac{y}{R}\right)^2+\cdots\right]$$

We add the constant WR, which is equivalent to changing the datum from $r = \infty$ to $r = R$:

$$V_g = WR\left[\frac{y}{R}-\left(\frac{y}{R}\right)^2+\cdots\right]$$

(*a*) First order approximation:

$$V_g = WR\left(\frac{y}{R}\right) = Wy \blacktriangleleft$$

[Equation 13.16]

(*b*) Second order approximation: $V_g = WR\left[\frac{y}{R}-\left(\frac{y}{R}\right)^2\right]$

$$V_g = Wy - \frac{Wy^2}{R} \blacktriangleleft$$

Chapter 13, Solution 93.

Use Conservation of energy and Conservation of angular momentum.

Conservation of angular momentum.

$$r_1 m v_{1\theta} = r_2 m v_{2\theta} \Rightarrow v_{\theta_2} = \frac{r_1 v_{\theta 1}}{r_2} = \frac{(0.2)(6)}{0.5}$$

$v_{\theta_2} = 2.4$ m/s ◀

Conservation of energy

$$T_1 + V_1 = T_2 + V_2 \quad (1)$$

At ① $T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(4\text{ kg})(6\text{ m/s})^2 = 72\text{ J}$

$$V_1 = \frac{1}{2} k x_1^2 = \frac{1}{2}(1500\text{ N/m})(0.2\text{ m} - 0.4\text{ m})^2 = 72\text{ J}$$

At ② $T_2 = \frac{1}{2} m v_{2\theta}^2 + \frac{1}{2} m v_{2r}^2 = \frac{1}{2}(4)(2.4)^2 + \frac{1}{2}(4) v_{2r}^2$

$$= 11.52 + 2v_{2r}^2$$

$$V_2 = \frac{1}{2} k x_2^2 = \frac{1}{2}(1500\text{ N/m})(0.5\text{ m} - 0.4\text{ m})^2 = 7.5\text{ J}$$

Substituting into (1) $72 + 30 = 11.52 + 2v_{2r}^2 + 7.5$

$$2v_{2r}^2 = 82.98$$

$v_{2r} = 6.44$ m/s ◀

Chapter 13, Solution 94.

Conservation of angular momentum.

$$r_1 \cancel{m} v_{1\theta} = r_{\max} \cancel{m} v_{2\theta}$$

so

$$v_{2\theta} = \frac{r_1 v_{1\theta}}{r_{\max}} = \frac{(0.2)(6)}{r_{\max}} = \frac{1.2}{r_{\max}}$$

Energy

$$T_1 + V_1 = T_2 + V_2 \qquad (1)$$

where,

At ① $$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(4\ \text{kg})(6\ \text{m/s})^2 = 72\ \text{J}$$

$$V_1 = \frac{1}{2} k x_1^2 = \frac{1}{2}(1500)(0.2 - 0.4)^2 = 30\ \text{J}$$

At ② $$T_2 = \frac{1}{2} m v_{2\theta}^2 + \cancelto{0}{\frac{1}{2} m v_{2r}^2} = \frac{1}{2}(4)\left(\frac{1.2}{r_{\max}}\right)^2 = \frac{288}{r_{\max}^2}$$

$$V_2 = \frac{1}{2} k x_2^2 = \frac{1}{2}(1500)(r_{\max} - 0.4)^2$$

Substituting into (1) $$72 + 30 = \frac{2.88}{r_{\max}^2} + 750(r_{\max} - 0.4)^2$$

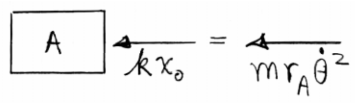
Solve by trial for $r_{\max}$ $\Rightarrow$ $r_{\max} = 0.760\ \text{m}$ ◀

Solve for $v_{2\theta}$

$$v_2 = \frac{1.2}{r_{\max}} = \frac{1.2}{0.760} \Rightarrow \qquad v_2 = 1.580\ \text{m/s}$$ ◀

Chapter 13, Solution 95.

Initial state



$$\overset{+}{\leftarrow} \quad \Sigma F = ma$$

$$kx_0 = mr_A\dot{\theta}^2, \qquad x_0 = \frac{mr_A\dot{\theta}^2}{k}$$

$$x_0 = \frac{(4/32.2)(3\text{ ft})(5\text{ rad/s})^2}{7\text{ lb/ft}} = 1.331\text{ ft}$$

Unstretched length = 4.5 ft + 1.331 ft = 5.831 ft

Conservation of angular momentum

(A) $\qquad h_A = r_A^2\dot{\theta} = (3)^2(5) = 45$ $\qquad$ (1)

(B) $\qquad h_B = r_B^2\dot{\theta} = (9)^2\dot{\theta} = (7.5)^2(5), \quad \dot{\theta} = 3.4722$ $\qquad$ (2)

(a) $\qquad$ Substitute into (1) gives $\qquad$ $r_A = 3.6$ ft ◀

(b) $\qquad$ $\dot{\theta} = 3.47$ rad/s ◀

Conservation of energy $\qquad T_0 + V_0 = T + V, \quad V_0 = \frac{1}{2}(7\text{ lb/ft})(1.331\text{ ft})^2 = 6.200\text{ ft}\cdot\text{lb}$

$$T_0 = \frac{1}{2}\left(\frac{4}{32.2}\right)\left[5(3)\right]^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)\left[5(7.5)\right]^2 = 101.320\text{ ft}\cdot\text{lb}$$

$$V = \frac{1}{2}(7)\left[(9-3.6)-5.831\right]^2 = 0.6501\text{ ft}\cdot\text{lb}$$

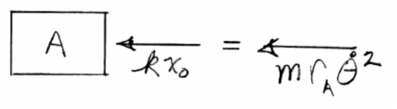
$$T = T_0 + V_0 - V$$

$$= 101.32 + 6.200 - 0.6501$$

(c) $\qquad$ $T = 106.9$ ft · lb◀

Chapter 13, Solution 96.

Initial state



$$\overset{+}{\leftarrow}\ \Sigma F = ma$$

$$kx_0 = mr_A\dot{\theta}^2$$

$$x_0 = \frac{mr_A\dot{\theta}^2}{k} = \frac{(4/32.2)(3)(5)^2}{7} = 1.331 \text{ ft}$$

Unstretched length = 4.5 + 1.331 = 5.831 ft

Conservation of energy

$$T_0 = \frac{1}{2}\left(\frac{4}{32.2}\right)\left[5(3)\right]^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)\left[5(7.5)\right]^2$$

$$= 101.320 \text{ ft}\cdot\text{lb}$$

$$V_0 = \frac{1}{2}(7)(1.331)^2 = 6.200 \text{ ft}\cdot\text{lb}$$

$$T + V = 100 + \frac{1}{2}(7)x_1^2 = 101.32 + 6.200$$

$$x_1 = \pm 1.4658 \text{ ft}$$

For compression:

$$(r_B - r_A) - 5.831 = -1.4658$$

Conservation of angular momentum

(A) $$h_A = r_A^2\dot{\theta} = (3)^2(5) = 45$$

(B) $$h_B = r_B^2\dot{\theta} = (7.5)^2(5) = 281.25$$

continued

$$r_B^2 = (2.5)^2 r_A^2, \quad r_B = 2.5 r_A$$

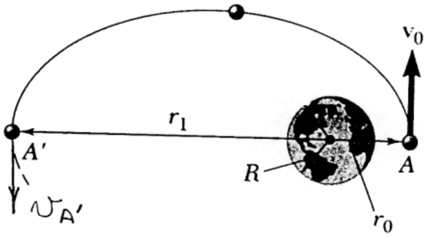
$$r_B - r_A - 5.831 = 2.5 r_A - r_A - 5.831 = 1.5 r_A - 5.831$$

(*a*) $\therefore \ 1.5 r_A - 5.831 = -1.4658$ $r_A = 2.91$ ft ◀

(*b*) $r_B = 2.5 r_A = 7.2753$ ft $r_B = 7.28$ ft ◀

(*c*) $r_A^2 \dot{\theta} = 45, \quad \dot{\theta} = 5.31$ rad/s $\dot{\theta} = 5.31$ rad/s ◀

Chapter 13, Solution 97.



$$R = 6370 \text{ km}$$

$$r_0 = 500 \text{ km} + 6370 \text{ km}$$

$$r_0 = 6870 \text{ km}$$

$$= 6.87 \times 10^6 \text{ m}$$

$$v_0 = 36{,}900 \text{ km/h}$$

$$= \frac{36.9 \times 10^6 \text{ m}}{3.6 \times 10^3 \text{ s}}$$

$$= 10.25 \times 10^3 \text{ m/s}$$

Conservation of angular momentum

$$r_0 m v_0 = r_1 m v_A, \qquad r_0 = r_{\min}, \; r_1 = r_{\max}$$

$$V_{A'} = \left(\frac{r_0}{r_1}\right) v_0 = \left(\frac{6.870 \times 10^6}{r_1}\right)\left(10.25 \times 10^3\right)$$

$$V_{A'} = \frac{70.418 \times 10^9}{r_1} \tag{1}$$

Conservation of energy

Point A

$$v_0 = 10.25 \times 10^3 \text{ m/s}$$

$$T_A = \frac{1}{2} m v_0^2 = \frac{1}{2} m \left(10.25 \times 10^3\right)^2$$

$$T_A = (m)\left(52.53 \times 10^6\right)(\text{J})$$

$$V_A = -\frac{GMm}{r_0} \qquad GM = gR^2 = \left(9.81 \text{ m/s}^2\right)\left(6.37 \times 10^6 \text{ m}\right)^2$$

$$GM = 398 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$r_0 = 6.87 \times 10^6 \text{ m}$$

continued

$$V_A = -\frac{\left(398\times10^{12}\text{ m}^3/\text{s}^2\right)m}{\left(6.87\times10^6\text{ m}\right)} = -57.93\times10^6\text{ m }(\text{J})$$

Point A'

$$T_{A'} = \frac{1}{2}mv_{A'}^2$$

$$V_{A'} = -\frac{GMm}{r_1} = -\frac{398\times10^{12}\text{ m}}{r_1}(\text{J})$$

$$T_A + V_A = T_{A'} + V_{A'}$$

$$52.53\times10^6\,\cancel{m} - 57.93\times10^6\,\cancel{m} = \frac{1}{2}\cancel{m}v_{A'}^2 - \frac{398\times10^{12}\,\cancel{m}}{r_1}$$

Substituting for $v_{A'}$ from (1)

$$-5.402\times10^6 = \frac{\left(70.418\times10^9\right)^2}{(2)(r_1)^2} - \frac{398\times10^{12}}{r_1}$$

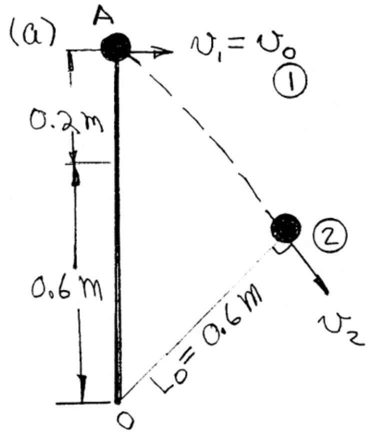
$$-5.402\times10^6 = \frac{\left(2.4793\times10^{21}\right)}{r_1^2} - \frac{398\times10^{12}}{r_1}$$

$$\left(5.402\times10^6\right)r_1^2 - \left(398\times10^{12}\right)r_1 + 2.4793\times10^{21} = 0$$

$$r_1 = 66.7\times10^6\text{ m},\ 6.87\times10^6\text{ m}$$

$r_{\max} = 66{,}700$ km ◀

Chapter 13, Solution 98.



The cord will not go slack if v_2 is perpendicular to the undeformed cord length, L_0, at ②

Conservation of angular momentum

$$0.8v_1 = 0.6v_2 \qquad v_2 = \frac{0.8}{0.6}v_1 = 1.333v_0$$

Conservation of energy

Point ① $\qquad v_1 = v_0 \qquad T_1 = \frac{1}{2}mv_0^2 = 0.35v_0^2$

$$V_1 = \frac{1}{2}k\left(L - L_0\right)^2 = \frac{1}{2}\left(150\text{ N/m}\right)\left(0.8\text{ m} - 0.6\text{ m}\right)^2$$

$$V_1 = 3\text{ J}$$

Point ② $\qquad T_2 = \frac{1}{2}mv_2^2 = 0.35v_2^2$

$$\Delta L = 0 \qquad V = 0 \qquad T_1 + V_1 = T_2 + V_2\text{:}\quad 0.35v_B^2 + 3 = 0.35v_2^2 + 0$$

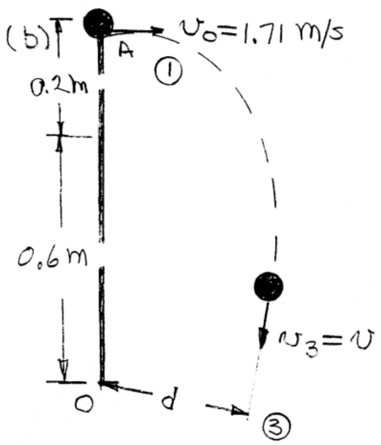
From conservation of angular momentum $\qquad v_2 = 1.3158v_B$

$$0.35v_0^2\left[\left(1.3158\right)^2 - 1\right] = 3$$

$$v_0^2 = \frac{\left(3\text{ J}\right)}{\left(0.35\text{ kg}\right)\left(0.7313\right)} = 11.72\text{ m}^2/\text{s}^2$$

$v_0 = 3.42$ m/s ◀

continued



The ball travels in a straight line after the cord goes slack.

Conservation of angular momentum

$$(0.8)(1.71) = dv$$

$$d = \frac{1.368}{v}$$

Conservation of energy

$$v_1 = 1.71\,\text{m/s}$$

Point ①

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.7\text{ kg})(1.71\text{ m/s})^2 = 1.0234\,\text{J}$$

$$V_1 = \frac{1}{2}k(L - L_0)^2 = \frac{1}{2}(150\text{ N/m})(0.8\text{ m} - 0.6\text{ m})^2 = 3\,\text{J}$$

Point ③ $$T_3 = \frac{1}{2}mv_3^2 = 0.35v^2$$

$$V_3 = 0$$

$$T_1 + V_1 = T_3 + V_3:\quad 1.0234 + 3 = 0.35v^2 + 0$$

$$v = 3.39\text{ m/s}$$

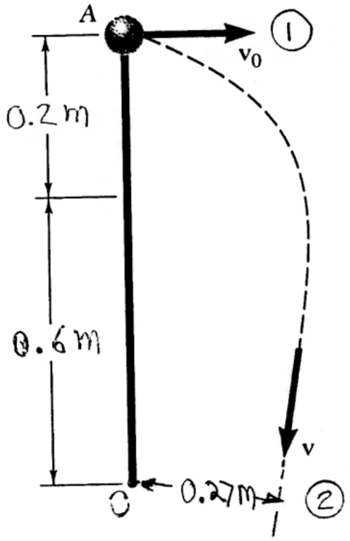
From conservation of momentum

$$d = \frac{1.368}{v} = \frac{1.368}{3.39} = 404\text{ mm}$$

$d = 404$ mm ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 99.



(*a*) Conservation of angular momentum: About *O*

$$0.8v_0 = 0.27v$$

$$v = 2.963v_0$$

Conservation of energy

Point ① $\quad v_1 = v_0 \quad T_1 = \frac{1}{2}mv_0^2 = 0.35v_0^2$

$$V_1 = \frac{1}{2}k\left(L_1 - L_0\right)^2 = \frac{1}{2}(150\text{ N/m})(0.8\text{ m} - 0.6\text{ m})^2$$

$$V_1 = 3\text{ J}$$

Point ② $\quad v_2 = v \quad T_2 = \frac{1}{2}mv^2 = 0.35v^2$

$$V_2 = 0 \;\left(\text{cord is slack}\right)$$

$$T_1 + V_1 = T_2 + V_2: \quad 0.35v_0^2 + 3 = 0.35v^2 + 0$$

From conservation of angular momentum, $\quad v = 3.125v_0$

$$0.35v_0^2\left[(3.125)^2 - 1\right] = 3$$

$$v_0^2 = \frac{(3\text{J})}{(0.35\text{ kg})(8.7656)}$$

$$v_0^2 = 0.9779\text{ m}^2/\text{s}^2$$

$$v_0 = 0.989\text{ m/s} ◀$$

(*b*) Maximum velocity occurs when the ball is at its minimum distance from *O*, (when $d = 0.27$ m)

$$v_m = 3.125v_0 = (3.125)(0.9889) = 3.09\text{ m/s}$$

$$v_m = 3.09\text{ m/s} ◀$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 100.

Conservation of angular momentum

$$mr_Av_A = mr_Bv_B \quad \Rightarrow v_B = \frac{r_A}{r_B}v_A \tag{1}$$

Conservation of energy

$$\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}mv_B^2 - \frac{GMm}{r_B} \tag{2}$$

Put (1) into (2) and solve for v_A

$$v_A^2 = \frac{2GMr_B}{r_A\left(r_B + r_A\right)} \tag{3}$$

Given data $\quad r_A = R + h_A = 1740 + 8 = 1748 \text{ km} = 1.748 \times 10^3 \text{ m}$

At B $\quad r_B = R + h_B = 1740 + 140 = 1880 \text{ km} = 1.880 \times 10^3 \text{ m}$

$$M_{\text{moon}} = 0.0123\, M_{\text{earth}}$$

$$\left(GM\right)_{\text{moon}} = 0.0123\left(GM\right)_{\text{earth}} = 0.0123\, gR^2$$

$$= 0.0123\left(9.81\right)\left(6.370 \times 10^6\right)^2 = 4.8961 \times 10^{12} \text{ m}^3/\text{s}^2$$

(*a*) Speed at *A*

$$v_A^2 = \frac{2\left(4.8961 \times 10^{12}\right)\left(1.88 \times 10^6\right)}{1.748 \times 10^6\left(1.880 \times 10^6 + 1.748 \times 10^6\right)} = 2.9029$$

$$v_A = 1703.8 \text{ m/s} \qquad v_A = 1704 \text{ m/s} \blacktriangleleft$$

(*b*) At *B*

$$v_B = \frac{r_A}{r_B} v_A = \frac{1748}{1880}(1703.8) = 1584.2 \text{ m/s}$$

The command module is in a circular orbit

At $r_B = 1.880 \times 10^6$ m

$$v_{\text{circ}} = \sqrt{\frac{GM}{r_B}} = \left[\frac{4.8961 \times 10^{12}}{1.880 \times 10^6}\right]^{\frac{1}{2}} = 1613.8 \text{ m/s}$$

$$\text{Relative velocity} = v_{circ} - v_B = (1613.8 - 1584.2) = 29.6 \text{ m/s}$$

Relative velocity = 29.6 m/s

Chapter 13, Solution 101.

From 13.100 $v_B = \frac{r_A}{r_B} v_A \quad (1) \qquad \text{and } v_A^2 = \frac{2GMr_B}{r_A(r_A + r_B)} \qquad (2)$

Given data $h_A = 264 \text{ km}; \; r_A = 6370 \text{ km} + 264 \text{ km} = 6634 \text{ km}$

$$= 6.634 \times 10^6 \text{ m}$$

$$h_B = 35780 \text{ km}; \; r_B = 6370 \text{ km} + 35780 \text{ km} = 42{,}150 \text{ km}$$

$$= 42.150 \times 10^6 \text{ m}$$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398.06 \times 10^{12}$$

Substitute into (2) $v_A^2 = \frac{2\left(398.06 \times 10^{12}\right)\left(42.150 \times 10^6\right)}{\left(6.634 \times 10^6\right)\left(42.150 \times 10^6 + 6.634 \times 10^6\right)}$

$$= 103.69 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_A = 10{,}183 \text{ m/s}$$

Substitute into (1) $v_B = \left(\frac{6634}{42{,}150}\right)(10{,}183) = 1602.7 \text{ m/s}$

At A we have a circular orbit

$$v_{circ} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.06 \times 10^{12}}{6.634 \times 10^6}} = 7746.2 \text{ m/s}$$

Relative velocity $v_A - v_{circ} = (10{,}183 - 7746.2)$

Relative velocity $= 2.44$ km/s ◀

At B $v_{circ} = \sqrt{\frac{GM}{r_B}} = \left[\frac{398.06 \times 10^{12}}{42.15 \times 10^6}\right]^{\frac{1}{2}} = 3073.1 \text{ m/s}$

Increase in velocity $= v_{circ} - v_B = 3073.1 - 1602.7 = 1470 \text{ m/s}$

Increase in $v = 1.470$ km/s ◀

Chapter 13, Solution 102.

M = Mass of the sun; $GM_E = gR^2$; $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

$$GM = 332.8(10)^2\left(32.2 \text{ ft/s}^2\right)\left(20.9088 \times 10^6 \text{ ft}\right)^2 = 4.6849 \times 10^{21} \text{ ft}^3/\text{s}^2$$

Circular orbits Earth $v_E = \sqrt{\dfrac{GM}{(93 \times 10^6)(5280)}} = 97,677 \text{ ft/s}$

Mars $v_M = \sqrt{\dfrac{GM}{(141.5 \times 10^6)(5280)}} = 79187 \text{ ft/s}$

Conservation of angular momentum

Elliptical orbit $v_A(93) = v_B(141.5)$

Conservation of energy

$$\frac{1}{2}v_A^2 - \frac{GM}{\left(93 \times 10^6\right)(5280)} = \frac{1}{2}v_B^2 - \frac{GM}{\left(141.5 \times 10^6\right)(5280)}$$

$$v_A = v_B\left(\frac{141.5}{93}\right) = 1.5215 v_B$$

$$\frac{1}{2}(1.5215)^2 v_B^2 - \frac{4.6849 \times 10^{21}}{\left(93 \times 10^6\right)(5280)} = \frac{1}{2}v_B^2 - \frac{4.6849 \times 10^{21}}{\left(141.5 \times 10^6\right)(5280)}$$

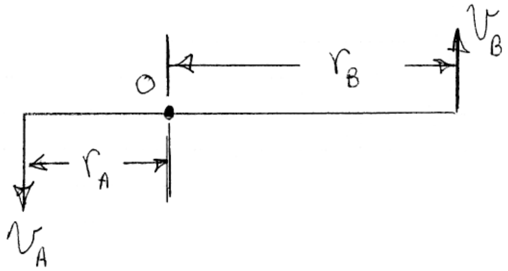
$$0.6575 v_B^2 = 3.270 \times 10^9$$

$$v_B = 70524 \text{ ft/s}; \quad v_A = 107,303 \text{ ft/s}$$

(*a*) Increase at *A*, $v_A - v_E = 107303 - 97677 = 9626 \text{ ft/s} = 1.823 \text{ mi/s}$ ◀

(*b*) Increase at *B*, $v_B - v_M = 79187 - 70524 = 8663 \text{ ft/s} = 1.641 \text{ mi/s}$ ◀

Chapter 13, Solution 103.



R = planet radius

$$r_A = R + h_A; \qquad r_B = R + h_B$$

Conservation of angular momentum

$$v_A\left(R + h_A\right) = v_B\left(R + h_B\right)$$

$$5\left(R + 1200 \text{ mi}\right) = 1.2\left(R + 16300 \text{ mi}\right)$$

$$5R + 6000 = 1.2R + 19560; \quad 3.8R = 13560$$

(a) $R = 3568.4$ mi $\qquad R = 3570$ mi ◀

Conservation of energy $\qquad m_s$ = mass of spacecraft

$$T_A + V_A = T_B + V_B; \quad \frac{1}{2}\cancel{m_s}v_A^2 - \frac{GM\cancel{m_s}}{\left(R + h_A\right)} = \frac{1}{2}\cancel{m_s}v_B^2 - \frac{\cancel{m_s}GM}{\left(R + h_B\right)}$$

$$\frac{\left(5(5280)\right)^2}{2} - \frac{GM}{\left(3568.4 + 1200\right)5280} = \frac{\left(1.2(5280)\right)^2}{2} - \frac{GM}{\left(3568.4 + 16300\right)\left(5280\right)}$$

$$348.48 \times 10^6 - \frac{GM}{25.177 \times 10^6} = 20.072 \times 10^6 - \frac{GM}{104.905 \times 10^6}$$

$$GM = 10.879 \times 10^{15} \text{ ft}^3/\text{s}^2$$

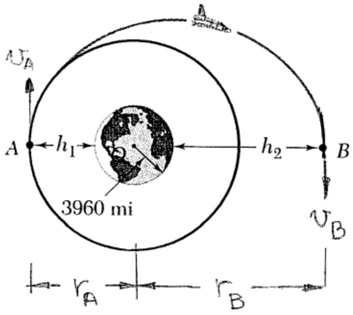
Using $\qquad G = 34.4 \times 10^{-9} \text{ ft}^4/\text{lb} \cdot \text{s}^4$

(b) $M = 316.26 \times 10^{21}$ slugs $\qquad M = 316 \times 10^{21}$ slugs ◀

(planet is Venus)

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 104.



Elliptical orbit between *A* and *B*

Conservation of angular momentum

$$mr_Av_A = mr_Bv_B$$

$$v_A = \frac{r_B}{r_A}v_B = \frac{7.170}{6.690}v_B$$

$$r_A = 6370\text{ km} + 320\text{ km} = 6690\text{ km}, \qquad r_A = 6.690\times10^6\text{ m}$$

$$v_A = 1.0718v_B \qquad (1)$$

$$r_B = 6370\text{ km} + 800\text{ km} = 7170\text{ km}, \qquad r_B = 7.170\times10^6\text{ m}$$

$$R = (6370\text{ km}) = 6.37\times10^6\text{ m}$$

Conservation of energy

$$GM = gR^2 = \left(9.81\text{ m/s}^2\right)\left(6.37\times10^6\text{ m}\right)^2 = 398.060\times10^{12}\text{ m}^3/\text{s}^2$$

Point A

$$T_A = \frac{1}{2}mv_A^2 \qquad V_A = -\frac{GMm}{r_A} = -\frac{\left(398.060\times10^{12}\right)m}{\left(6.690\times10^6\right)}$$

$$V_A = 59.501\times10^6\text{m}$$

Point B

$$T_B = \frac{1}{2}mv_B^2 \qquad V_B = -\frac{GMm}{r_B} = -\frac{\left(398.060\times10^{12}\right)m}{\left(7.170\times10^6\right)}$$

$$V_B = 55.5\times10^6\text{m}$$

$$T_A + V_A = T_B + V_B$$

continued

$$\frac{1}{2}mv_A^2 - 59.501\times10^6\text{m} = \frac{1}{2}mv_B^2 - 55.5\times10^6\text{m}$$

$$v_A^2 - v_B^2 = 8.002\times10^6$$

From (1) $\qquad v_A = 1.0718v_B \qquad v_B^2\left[(1.0718)^2 - 1\right] = 8.002\times10^6$

$$v_B^2 = 53.79\times10^6\ \text{m}^2/\text{s}^2, \qquad v_B = 7334\ \text{m/s}$$

$$v_A = (1.0718)(7334\ \text{m/s}) = 7861\ \text{m/s}$$

Circular orbit at *A* and *B*

(Equation 12.44)

$$(v_A)_C = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.060\times10^{12}}{6.690\times10^6}} = 7714\ \text{m/s}$$

$$(v_B)_C = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{398.060\times10^{12}}{7.170\times10^6}} = 7451\ \text{m/s}$$

(*a*) Increases in speed at *A* and *B*

$$\Delta v_A = v_A - (v_A)_C = 7861 - 7714 = 147\ \text{m/s} \blacktriangleleft$$

$$\Delta v_B = (v_B)_C - v_B = 7451 - 7334 = 117\ \text{m/s} \blacktriangleleft$$

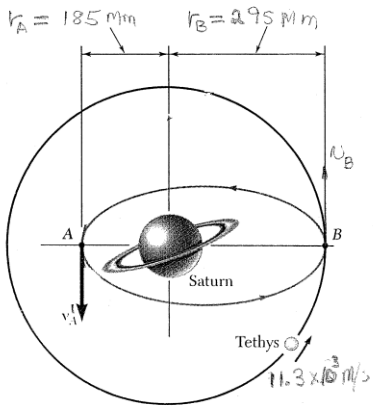
(*b*) Total energy per unit mass

$$E/m = \frac{1}{2}\left[(v_A)^2 - (v_A)_C^2 + (v_B)_C^2 - (v_B)^2\right]$$

$$E/m = \frac{1}{2}\left[(7861)^2 - (7714)^2 + (7451)^2 - (7334)^2\right]$$

$$E/m = 2.01\times10^6\ \text{J/kg} \blacktriangleleft$$

Chapter 13, Solution 105.



(*a*)

$$r_A = 185 \times 10^6 \text{ m}$$

$$r_B = 295 \times 10^6 \text{ m}$$

$v'_A =$ Speed of spacecraft in the elliptical orbit after its speed has been decreased

Elliptical orbit between A and B conservation of energy

Point A $\qquad T_A = \frac{1}{2} m v_A^2, \qquad V_A = -\frac{GM_{\text{sat}} m}{r_A}$

$$M_{\text{sat}} = \text{mass of saturn}$$

Determine GM_{sat} from the speed of Tethys in its circular orbit.

(Equation 12.44) $\qquad v_{\text{circ}} = \sqrt{\frac{GM_{\text{sat}}}{r}} \qquad GM_{\text{sat}} = r_B v_{\text{circ}}^2$

$$GM_{\text{sat}} = \left(295 \times 10^6 \text{ m}\right)\left(11.3 \times 10^3 \text{ m/s}\right)^2 = 37.67 \times 10^{15} \text{ m}^3/\text{s}^2$$

$$V_A = -\frac{\left(37.67 \times 10^{15} \text{ m}^3/\text{s}^2\right) m}{\left(185 \times 10^6 \text{ m}\right)} = -0.2036 \times 10^9 \text{m}$$

Point B

$$T_B = \frac{1}{2} m v_B^2 \qquad V_B = -\frac{GM_{\text{sat}} m}{r_B} = -\frac{\left(37.67 \times 10^{15} \text{ m}^3/\text{s}^2\right) m}{\left(295 \times 10^6 \text{m}\right)}$$

$$V_B = -0.1277 \times 10^9$$

$$T_A + V_A = T_B + V_B;$$

$$\frac{1}{2} \cancel{m} v_A^2 - 0.2036 \times 10^9 \cancel{m} = \frac{1}{2} \cancel{m} v_B^2 - 0.1277 \times 10^9 \cancel{m}$$

$$v_A'^2 - v_B^2 = 0.1518 \times 10^9$$

continued

Conservation of angular momentum

$$r_A m v'_A = r_B m v_B \qquad v_B = \frac{r_A}{r_B} v'_A = \frac{185 \times 10^6}{295 \times 10^6} v'_A = 0.6271 v'_A$$

$$v'^2_A \left[1 - (0.6271)^2 \right] = 0.1518 \times 10^9, \qquad v'_A = 15817 \text{ m/s}$$

$$\Delta v_A = v_A - v'_A = 21000 - 15817 = 5183 \text{ m/s} = 5.18 \text{ km/s} \blacktriangleleft$$

(*b*)

$$v_B = \frac{r_A}{r_B} v'_A = (0.6271)(15817) = 9919 \text{ m/s},$$

$$v_B = 9.92 \text{ km/s} \blacktriangleleft$$

Chapter 13, Solution 106.

Conservation of energy

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_B} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$$

So $$v_A^2 = v_0^2 - \frac{2GM}{r_B}\left(1 - \frac{r_B}{r_A}\right) \qquad (1)$$

Given

$$R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$$

$$GM = gR^2 = (9.81)\left(6.37 \times 10^6\right)^2 = 398 \times 10^{12}$$

$$r_A = 6370 + 360 = 6730 \text{ km} = 6.73 \times 10^6 \text{ m}$$

$$r_B = 6370 + 60 = 6430 \text{ km} = 6.43 \times 10^6 \text{ m}$$

Substitute into (1)

$$v_A^2 = v_0^2 - \frac{2\left(398 \times 10^{12}\right)}{6.43 \times 10^6}\left(1 - \frac{6.43 \times 10^6}{6.73 \times 10^6}\right)$$

$$v_A^2 = v_0^2 - 5.518 \times 10^6 \qquad (2)$$

We need another equation → conservation of angular momentum

$$r_B\, mv \sin\phi = r_A\, m\, v_A$$

$$v_A = \frac{r_B\, v_0 \sin\phi}{r_A} = \left(\frac{6.43 \times 10^6}{6.73 \times 10^6}\right) v_0 \sin 50^\circ$$

$$v_A = 0.7319\, v_0$$

Substitute into (2)

$$\left(0.7319\, v_0\right)^2 = v_0^2 - 5.518 \times 10^6$$

$$0.46433 v_0^2 = 5.518 \times 10^6$$

$$v_0 = 3477 \text{ m/s}$$

$v_0 = 3450$ m/s ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 107.

Let $E = \dfrac{T+V}{m} = \text{constant} = \dfrac{1}{2}v_A^2 - \dfrac{GM}{r_A}$

$$v_A = \left(30{,}000\ \frac{\text{km}}{\text{hr}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)\left(\frac{1}{3600\ \text{s/hr}}\right)$$

$$v_A = 8333.33\ \text{m/s}$$

$$GM = gR^2 = \left(9.81\ \text{m/s}^2\right)\left(6.37\times10^6\ \text{m}\right)^2$$

$$GM = 398.059\times10^{12}\ \text{m}^3/\text{s}^2$$

$$r_A = 4.3\times10^6\ \text{m} + R = 4.3\times10^6\ \text{m} + 6.37\times10^6\ \text{m}$$

$$r_A = 10.67\times10^6\ \text{m}$$

$$\therefore E = \frac{1}{2}(8333.3)^2 - \frac{398.059\times10^{12}}{10.67\times10^6}$$

$$E = -2584.19\times10^3\ \text{m}^2/\text{s}^2$$

$$h/m = \text{constant} = v_A r_A \sin 60°$$

$$h/m = (8333.33\ \text{m/s})\left(10.67\times10^6\ \text{m}\right)\left(\sqrt{3}/2\right)$$

$$h/m = 77.0041\times10^9\ \text{m}^2/\text{s}$$

At min or max altitude, $h/m = rv,\ \therefore\ v = \left(\dfrac{h/m}{r}\right)$

Eliminate

$$v = \left(\frac{h/m}{r}\right):\ E = \frac{1}{2}v^2 - \frac{GM}{R} = \frac{1}{2}\left(\frac{h/m}{r}\right)^2 - \frac{GM}{r}$$

Multiply by r^2: $Er^2 = \dfrac{1}{2}\left(\dfrac{h}{m}\right)^2 - GMr$

And rearrange $$Er^2 + (GM)r - \frac{1}{2}\left(\frac{h}{m}\right)^2 = 0$$

Quadratic formula for minimum r: $$r_{\min} = \frac{-GM + \sqrt{(GM)^2 + 2E(h/m)^2}}{2E}$$

$$r_{\min} = \frac{-398.06\times10^{12} + \sqrt{(398.06)^2\times10^{24} + 2\left(-2584.2\times10^3\right)(77.004)^2\times10^{18}}}{2\left(-2584.2\times10^3\right)}$$

$$r_{\min} = \frac{-398.06\times10^{12} + 357.50\times10^{12}}{2\left(-2584.2\times10^3\right)}$$

$$r_{\min} = 7.848\times10^6 \text{ m}$$

$$\text{Minimum altitude} = 7.848\times10^6 - R$$

$$= 7.848\times10^6 \text{ m} - 6.37\times10^6 \text{ m}$$

$$= 1.478\times10^6 \text{ m}$$

Minimum altitude = 1478 km ◀

Chapter 13, Solution 108.

At A: $$h_A = vr = \left[(6.5)(5280)\right]\left[(3960\text{ mi} + 567\text{ mi})(5280\text{ ft/mi})\right]$$

$$h_A = 820.336\times 10^9\ \text{ft}^2/\text{s}$$

$$R = (3960\text{ mi})(5280\text{ ft/mi}) = 20.9088\times 10^6\ \text{ft}$$

$$\frac{1}{m}(T_A + V_A) = \frac{1}{2}v^2 - \frac{GM}{r}$$

$$= \frac{1}{2}\left[(6.5)(5280)\right]^2 - \frac{(32.2)\left[20.9088\times 10^6\right]^2}{\left[(3960+567)(5280)\right]} \cong 0$$

Parabolic orbit

At B: $$\frac{1}{m}(T_B + V_B) = \frac{1}{2}v_B^2 - \frac{GM}{r_B} = 0$$

$$\frac{1}{2}v_B^2 = \frac{32.2\left[20.9088\times 10^6\right]^2}{\left[(3960+5190)(5280)\right]}$$

$$v_B^2 = 582.76\times 10^6;\quad v_B = 24140\text{ ft/s}$$

(*a*) $v_B = 4.57$ mi/s ◀

$$h_B = v_B \sin\phi_B\, r_B = 820.336\times 10^9$$

$$\sin\phi_B = \frac{820.336\times 10^9}{24140\left[(3960+5190)(5280)\right]}$$

$$= 0.7034$$

(*b*) $\phi_B = 44.7°$ ◀

Chapter 13, Solution 109.

$$h_A = v_A r_A, \quad r_A = 3960 \text{ mi} + 377 \text{ mi} = 4337 \text{ mi} = 22.8994 \times 10^6 \text{ ft}$$

$$v_C = (2.97 \text{ mi/s})(5280 \text{ ft/mi}) = 15681.6 \text{ ft/s}$$

$$R = (3960)(5280) = 20.9088 \times 10^6 \text{ ft}$$

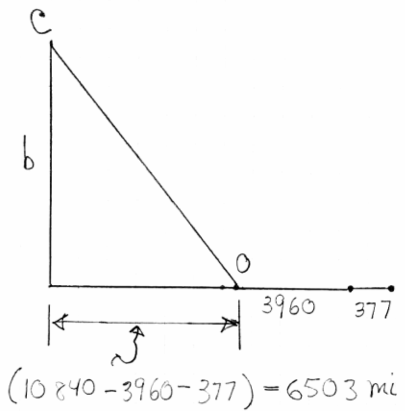
$$h_C = v_C\, b = 15681.6\, b = v_A\left[22.8994 \times 10^6\right] \qquad (1)$$

$$\frac{1}{m}(T_A + V_A) = \frac{v_A^2}{2} - \frac{GM}{22.8994 \times 10^6}$$

$$GM = 32.2\left[20.9088 \times 10^6\right]^2$$

$$= \frac{v_A^2}{2} - \frac{GM}{22.8994 \times 10^6} \qquad (2)$$

Two equations in two unknowns: v_A, b.



Solve ;

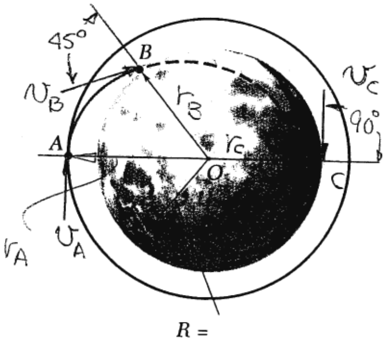
$$v_A = 31361 \text{ ft/s}$$

$$b = 45.8 \times 10^6 \text{ ft}$$

(*a*) $v_A = 5.94$ mi/s ◀

(*b*) $b = 8670$ mi ◀

Chapter 13, Solution 110.



$$r_A = 1080 + 87 = 1167 \text{ mi} = 6.1618 \times 10^6 \text{ ft}$$

$$r_C = R = 1080 \text{ mi} = 5.7024 \times 10^6 \text{ ft}$$

$$GM_{\text{moon}} = 0.0123 \qquad GM_E = 0.0123 g R_E^2$$

$$= 0.0123(32.2)(3960 \times 5280)^2$$

$$= 1.7315 \times 10^{14} \text{ ft}^3/\text{s}^2$$

At 87 mi: $$v_{\text{circ}} = \sqrt{\frac{GM_{\text{moon}}}{r_A}} = 5301.0 \text{ ft/s}$$

(*a*) An elliptic trajectory between *A* and *C*, where the lem is just tangent to the surface of the moon, will give the smallest reduction of speed at *A* which will cause impact.

$$T_A = \frac{1}{2} m v_A^2 \qquad V_A = -\frac{GM_m m}{r_A} = -28.101 \times 10^6 \text{m}$$

$$T_C = \frac{1}{2} m v_C^2 \qquad V_C = -\frac{GM_m m}{r_C} = -30.364 \times 10^6 \text{m}$$

$$T_A + V_A = T_C + V_C: \ \frac{1}{2} m v_A^2 - 28.101 \times 10^6 \text{m}$$

$$= \frac{1}{2} m v_C^2 - 30.364 \times 10^6 \text{m}$$

$$v_A^2 = v_C^2 - 4.526 \times 10^6 \qquad (1)$$

Conservation of angular momentum: $r_A m v_A = r_C m v_C$

$$v_C = \frac{r_A}{r_C} v_A = \frac{6.1618}{5.7024} v_A = 1.0806 v_A$$

$$v_A^2 = (1.0806 v_A)^2 - 4.526 \times 10^6 \Rightarrow v_A = 5195.1 \text{ ft/s}$$

$$\Delta v_A = (v_A)_{\text{circ}} - v_A = 5343.9 - 5195.1 = 148.8$$

$\Delta v_A = 148.8$ ft/s ◀

(*b*) Conservation of energy (A and B)

Since $r_B = r_C$ conservation of energy is the same as between *A* and *C*

Conservation of angular momentum:

$$r_A m v_A = r_B m v_B \sin\phi; \qquad \phi = 45°$$

$$v_B = \frac{r_A}{r_B} \frac{v_A}{\sin 45°} = \left(\frac{6.1618}{5.7024}\right)\left(\frac{V_A}{0.70711}\right) = 1.5281 v_A$$

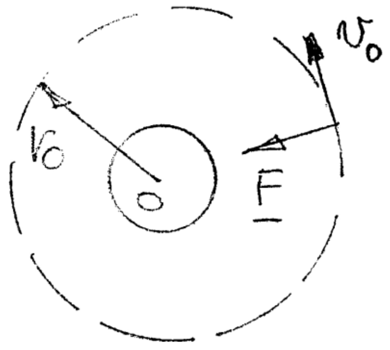
From (1)

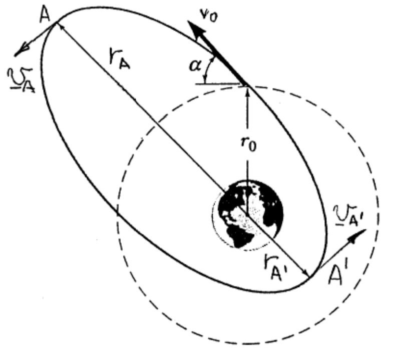
$$v_A^2 = (1.5281 v_A)^2 - 4.526 \times 10^6 \Rightarrow v_A = 1841.4 \text{ ft/s}$$

$$\Delta v_A = (v_A)_{\text{circ}} - v_A = 5343.9 - 1841.4 = 3487.3$$

$\Delta v_A = 3503$ ft/s ◀

Chapter 13, Solution 111.





For circular orbit of radius r_0

$$F = ma_n \qquad \frac{GMm}{r_0^2} = m\frac{v_0^2}{r_0}$$

$$v_0^2 = \frac{GM}{r_0}$$

But v_0 forms an angle α with the intended circular path

For elliptic orbit

Conservation of angular momentum

$$r_0 m v_0 \cos\alpha = r_A m v_A$$

$$v_A = \left(\frac{r_0}{r_A}\cos\alpha\right)v_0 \qquad (1)$$

Conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{GMm}{r_0} = \frac{1}{2}mv_A^2 = \frac{GMm}{r_A}$$

$$v_0^2 - v_A^2 = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

Substitute for v_A from (1)

$$v_0^2\left[1 - \left(\frac{r_0}{r_A}\right)^2\cos^2\alpha\right] = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

But $v_0^2 = \dfrac{GM}{r_0}$ thus $\qquad 1 - \left(\dfrac{r_0}{r_A}\right)^2\cos^2\alpha = 2\left(1 - \dfrac{r_0}{r_A}\right)$

$$\cos^2\alpha\left(\frac{r_0}{r_A}\right)^2 - 2\left(\frac{r_0}{r_A}\right) + 1 = 0$$

Solving for $\frac{r_0}{r_A}$

$$\frac{r_0}{r_A} = \frac{+2 \pm \sqrt{4 - 4\cos^2\alpha}}{2\cos^2\alpha} = \frac{1 \pm \sin\alpha}{1 - \sin^2\alpha}$$

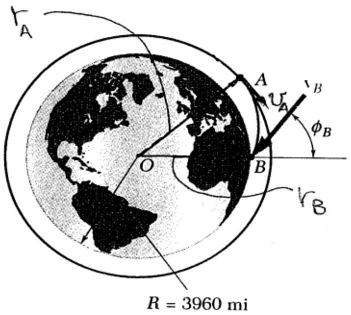
$$r_A = \frac{(1+\sin\alpha)(1-\sin\alpha)}{1 \pm \sin\alpha} r_0 = (1 \mp \sin\alpha) r_0$$

$\nwarrow$ also valid for point A'

Thus

$$r_{\text{max}} = (1+\sin\alpha) r_0 \qquad r_{\text{min}} = (1-\sin\alpha) r_0 \blacktriangleleft$$

Chapter 13, Solution 112.



$$r_A = 6370 \text{ km} + 362 \text{ km} = 6732 \text{ km}$$

$$r_A = 6.732 \times 10^6 \text{ m}$$

$$r_B = 6370 \text{ km} + 64.4 \text{ km} = 6434.4 \text{ km}$$

$$r_B = 6.4344 \times 10^6 \text{ m}$$

$$R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$$

$$GM = gR^2 = \left(9.81 \text{ m/s}^2\right)\left(6.37 \times 10^6 \text{ m}\right)^2$$

$$GM = 398.06 \times 10^9 \text{ m}^3\text{/s}^2$$

Conservation of energy

$$T_A = \frac{1}{2} m v_A^2 \qquad V_A = -\frac{GMm}{r_A} = -\frac{398.06 \times 10^9 m}{6.732 \times 10^6} = -59.130 \times 10^6 \text{m}$$

$$T_B = \frac{1}{2} m v_B^2 \qquad V_B = -\frac{GMm}{r_B} = -\frac{398.06 \times 10^9 m}{6.4344 \times 10^6} = -61.86 \times 10^6 \text{m}$$

$$T_A + V_A = T_B + V_B: \quad \frac{1}{2}\cancel{m} v_A^2 - 59.130 \times 10^6 \cancel{m} = \frac{1}{2}\cancel{m} v_B^2 - 61.86 \times 10^6 \cancel{m}$$

$$v_A^2 = v_B^2 - 5.4609 \times 10^6 \qquad (1)$$

Conservation of angular momentum

$$r_A m v_A = r_B m v_B \sin\phi_B$$

$$v_B = \frac{(r_A)v_A}{(r_B)(\sin\phi_B)} = \frac{6732}{6434.4}\left(\frac{1}{\sin 60^\circ}\right)v_A \qquad v_B = 1.208 v_A \qquad (2)$$

Substitute v_B from (2) in (1)

$$v_A^2 = (1.208v_A)^2 - 5.4609\times 10^6; \;\; v_A^2\left[(1.208)^2 - 1\right] = 5.4609\times 10^6$$

$$v_A^2 = 11.8905\times 10^6 \text{ m}^2/\text{s}^2$$

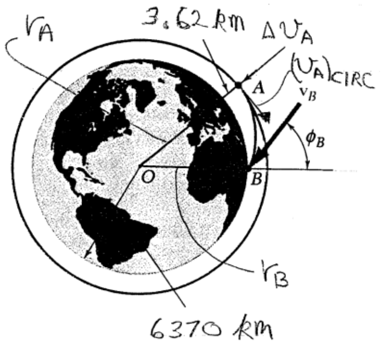
$$v_A = 3.448 \text{ km/s}$$

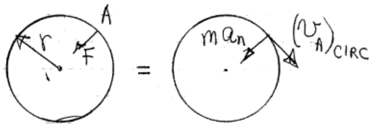
(*a*) $v_A = 3.45$ km/s ◀

From (2) $v_B = 1.208 v_A = 1.208(3.45 \text{ km/s}) = 4.1655 \text{ km/s}$

(*b*) $v_B = 4.17$ km/s ◀

Chapter 13, Solution 113.





$$r_A = 6370 \text{ km} + 362 \text{ km}$$

$$= 6732 \text{ km}$$

$$r_B = 6370 \text{ km} + 64.4 \text{ km}$$

$$= 6434.4 \text{ km}$$

$$GM = gR^2 = \left(9.81 \text{ m/s}^2\right)\left[6.37 \times 10^6 \text{ m}\right]^2$$

$$GM = 0.39806 \times 10^{15} \text{ m}^3/\text{s}^2$$

Velocity in circular orbit at 362 km altitude

$$F = \frac{GMm}{r_A^2}$$

$$a_n = \frac{m(v_A)_{\text{circ}}^2}{r_A^2}$$

Newton's second law

$$F = ma_n \qquad \frac{GMm}{r_A^2} = \frac{m(v_A)_{\text{circ}}^2}{r_A}$$

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{0.39806 \times 10^{15}}{6.732 \times 10^6}} = 7.69 \times 10^3 \text{ m/s}$$

Energy expenditure

From Problem 13.112, $v_A = 3.448 \times 10^3$ m/s

Energy, $$\Delta E_{112} = \frac{1}{2}m\left(v_A\right)^2_{\text{circ}} - \frac{1}{2}mv_A^2$$

$$\Delta E_{112} = \frac{1}{2}m\left(7.690\times10^3\right)^2 - \frac{1}{2}m\left(3.448\times10^3\right)^2$$

$$\Delta E_{112} = 23.624\times10^6\text{m J}$$

$$\Delta E_{113} = 0.50\Delta E_{112} = \frac{\left(23.624\times10^6\text{m}\right)}{2}\text{ J}$$

Thus, additional kinetic energy at A is,

$$\frac{1}{2}m\left(\Delta v_A\right)^2 = \Delta E_{113} = \frac{\left(23.624\times10^6\text{m}\right)}{2} \qquad (1)$$

Conservation of energy between A and B

$$T_A = \frac{1}{2}m\left[\left(v_A\right)^2_{\text{circ}} + \left(\Delta v_A\right)^2\right] \qquad V_A = -\frac{GMm}{r_A}$$

$$T_B = \frac{1}{2}mv_B^2 \qquad V_B = -\frac{GMm}{r_B}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m\left(7.690\times10^3\right)^2 + \frac{23.624\times10^6\text{m}}{2} - \frac{0.3980\times10^{15}\text{m}}{6.732\times10^6}$$

$$= \frac{1}{2}mv_B^2 - \frac{0.39806\times10^{15}\text{m}}{6.434\times10^6}$$

$$v_B^2 = 59.136\times10^6 + 23.624\times10^6 - 118.26\times10^6 + 123.74\times10^6$$

$$v_B^2 = 88.240\times10^6$$

$$v_B = 9.39\text{ km/s} \blacktriangleleft$$

Conservation of angular momentum between A and B

$$r_A m\left(v_A\right)_{\text{circ}} = r_B m v_B \sin\phi_B$$

$$\sin\phi_B = \left(\frac{r_A}{r_B}\right)\frac{\left(v_A\right)_{\text{circ}}}{\left(v_B\right)} = \frac{(6732)}{(6434.4)}\frac{\left(7.690\times10^3\right)}{\left(9.394\times10^3\right)} = 0.8565$$

$$\phi_B = 58.9^\circ \blacktriangleleft$$

Chapter 13, Solution 114.

Conservation of angular momentum

$$r_A m v_A = r_P m v_P$$

$$v_A = \frac{r_P}{r_A} v_P \qquad (1)$$

Conservation of energy

$$\frac{1}{2} m v_P^2 - \frac{GMm}{r_P} = \frac{1}{2} m v_A^2 - \frac{GMm}{r_A} \qquad (2)$$

Substituting for v_A from (1) into (2)

$$v_P^2 - \frac{2GM}{r_P} = \left(\frac{r_P}{r_A}\right)^2 v_P^2 - \frac{2GM}{r_A}$$

$$\left(1 - \left(\frac{r_P}{r_A}\right)^2\right) v_P^2 = 2GM\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$$

$$\frac{r_A^2 - r_P^2}{r_A^2} v_P^2 = 2GM \frac{r_A - r_P}{r_A r_P}$$

With $$r_A^2 - r_P^2 = (r_A - r_P)(r_A + r_P)$$

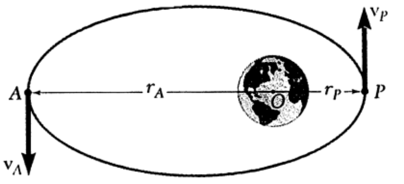
$$v_P^2 = \frac{2GM}{r_A + r_P}\left(\frac{r_A}{r_P}\right) \quad (3) \blacktriangleleft$$

Exchanging subscripts *P* and *A*

$$v_A^2 = \frac{2GM}{r_A + r_P}\left(\frac{r_P}{r_A}\right) \quad (\text{QED}) \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 115.



See solution to Problem 13.113 (above) for derivation of Equation (3)

$$v_P^2 = \frac{2GM}{(r_A + r_P)}\frac{r_A}{r_P}$$

Total energy at point P is

$$E = T_P + V_P = \frac{1}{2}mv_P^2 - \frac{GMm}{r_P}$$

$$= \frac{1}{2}\left[\frac{2GMm}{(r_A + r_0)}\frac{r_A}{r_P}\right] - \frac{GMm}{r_P}$$

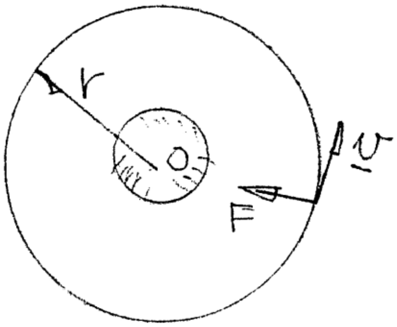
$$= GMm\left[\frac{r_A}{r_P(r_A + r_P)} - \frac{1}{r_P}\right]$$

$$= GMm\frac{(r_A - r_A - r_P)}{r_P(r_A + r_P)}$$

$$E = -\frac{GMm}{r_A + r_P} \blacktriangleleft$$

Note: Recall that gravitational potential of a satellite is defined as being zero at an infinite distance from the earth.

Chapter 13, Solution 116.



(*a*) For a *circular* orbit of radius r

$$F = ma_n: \quad \frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM}{r}$$

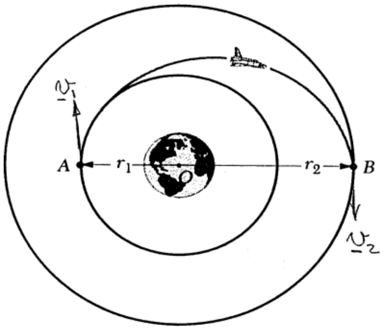
$$E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r} \qquad (1)$$

Thus ΔE required to pass from circular orbit of radius r_1 to circular orbit of radius r_2 is

$$\Delta E = E_1 - E_2 = -\frac{1}{2}\frac{GMm}{r_1} + \frac{1}{2}\frac{GMm}{r_2}$$

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1r_2} \qquad (2)\ \text{(Q.E.D.)}$$

(*b*) For an *elliptic orbit* we recall Equation (3) derived in Problem 13.113 $(\text{with } v_P = v_1)$



$$v_1^2 = \frac{2Gm}{(r_1 + r_2)}\frac{r_2}{r_1}$$

At point A: Initially spacecraft is in a circular orbit of radius r_1

$$v_{\text{circ}}^2 = \frac{GM}{r_1}$$

$$T_{\text{circ}} = \frac{1}{2}mv_{\text{circ}}^2 = \frac{1}{2}m\frac{GM}{r_1}$$

After the spacecraft engines are fired and it is placed on a semi-elliptic path *AB*, we recall

$$v_1^2 = \frac{2GM}{(r_1 + r_2)} \cdot \frac{r_2}{r_1}$$

And

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)}$$

At point *A*, the increase in energy is

$$\Delta E_A = T_1 - T_{\text{circ}} = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)} - \frac{1}{2}m\frac{GM}{r_1}$$

$$\Delta E_A = \frac{GMm(2r_2 - r_1 - r_2)}{2r_1(r_1 + r_2)} = \frac{GMm(r_2 - r_1)}{2r_1(r_1 + r_2)}$$

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\left[\frac{GMm(r_2 - r_1)}{2r_1r_2}\right]$$

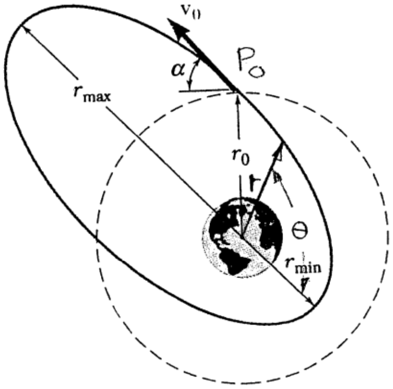
Recall Equation (2):

$$\Delta E_A = \frac{r_2}{(r_1 + r_2)}\Delta E \qquad (\text{Q.E.D})$$

A similar derivation at point *B* yields,

$$\Delta E_B = \frac{r_1}{(r_1 + r_2)}\Delta E \qquad (\text{Q.E.D})$$

Chapter 13, Solution 117.



If the point of intersection P_0 of the circular and elliptic orbits is at an end of the minor axis, then v_0 is parallel to the major axis. This will be the case only if $\alpha + 90° = \theta_0$, that is if $\cos\theta_0 = -\sin\alpha$ we must therefore prove that

$$\cos\theta_0 = -\sin\alpha \qquad (1)$$

We recall from Equation (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \qquad (2)$$

When $\theta = 0$, $\quad r = r_{\text{min}}$ and $r_{\text{min}} = r_0(1 - \sin\alpha)$

$$\frac{1}{r_0(1 - \sin\alpha)} = \frac{GM}{h^2} + C \qquad (3)$$

For $\theta = 180°$, $\quad r = r_{\text{max}} = r_0(1 + \sin\alpha)$

$$\frac{1}{r_0(1 + \sin\alpha)} = \frac{GM}{h^2} - C \qquad (4)$$

Adding (3) and (4) and dividing by 2:

$$\frac{GM}{h^2} = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} + \frac{1}{1+\sin\alpha}\right) = \frac{1}{r_0\cos^2\alpha}$$

Subtracting (4) from (3) and dividing by 2:

$$C = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} - \frac{1}{1+\sin\alpha}\right) = \left(\frac{1}{2r_0}\right)\frac{2\sin\alpha}{1-\sin^2\alpha}$$

$$C = \frac{\sin\alpha}{r_0\cos^2\alpha}$$

Substitute for $\dfrac{GM}{h^2}$ and C into Equation (2)

$$\frac{1}{r} = \frac{1}{r_0\cos^2\alpha}(1 + \sin\alpha\cos\theta) \qquad (5)$$

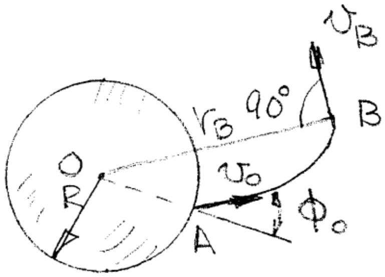
Letting $r = r_0$ and $\theta = \theta_0$ in Equation (5), we have

$$\cos^2\alpha = 1 + \sin\alpha\cos\theta_0$$

$$\cos\theta_0 = \frac{\cos^2\alpha - 1}{\sin\alpha} = -\frac{\sin^2\alpha}{\sin\alpha} = -\sin\alpha$$

This proves the validity of Equation (1) and thus P_0 is an end of the minor axis of the elliptic orbit.

Chapter 13, Solution 118.



(*a*)

$$r_A = R$$

Conservation of angular momentum

$$Rmv_0\sin\phi_0 = r_Bmv_B$$

$$r_B = R + \alpha R = (1+\alpha)R$$

$$v_B = \frac{Rv_0\sin\phi_0}{(1+\alpha)R} = \frac{v_0\sin\phi_0}{(1+\alpha)} \qquad (1)$$

Conservation of Energy

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv_B^2 - \frac{GMm}{(1+\alpha)R}$$

$$v_0^2 - v_B^2 = \frac{2GMm}{R}\left(1 - \frac{1}{1+\alpha}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

Substitute for v_B from (1)

$$v_0^2\left(1 - \frac{\sin^2\phi_0}{(1+\alpha)^2}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

From Equation (12.43): $\qquad v_{\text{esc}}^2 = \dfrac{2GM}{R}$

$$v_0^2\left(1 - \frac{\sin^2\phi_0}{(1+\alpha)^2}\right) = v_{\text{esc}}^2\left(\frac{\alpha}{1+\alpha}\right)$$

$$\frac{\sin^2\phi_0}{(1+\alpha)^2} = 1 - \left(\frac{v_{\text{esc}}}{v_0}\right)^2\frac{\alpha}{1+\alpha} \qquad (2)$$

$$\sin\phi_0 = (1+\alpha)\sqrt{1 - \frac{\alpha}{1+\alpha}\left(\frac{v_{\text{esc}}}{v_0}\right)^2} \qquad \text{Q.E.D.}$$

(*b*) Allowable values of v_0 $\left(\text{for which maximum altitude } = \alpha R\right)$

$$0 \le \sin^2 \phi_0 \le 1$$

For $\sin\phi_0 = 0$, from (2)

$$0 = 1 - \left(\frac{v_{\text{esc}}}{v_0}\right)^2 \frac{\alpha}{1 + \alpha}$$

$$v_0 = v_{\text{esc}}\sqrt{\frac{\alpha}{1 + \alpha}}$$

For $\sin\phi_0 = 1$, from (2)

$$\frac{1}{(1 + \alpha)^2} = 1 - \left(\frac{v_{\text{esc}}}{v_0}\right)^2 \frac{\alpha}{1 + \alpha}$$

$$\left(\frac{v_{\text{esc}}}{v_0}\right)^2 = \frac{1}{\alpha}\left(1 + \alpha - \frac{1}{1 + \alpha}\right) = \frac{1 + 2\alpha + \alpha^2 - 1}{\alpha(1 + \alpha)} = \frac{2 + \alpha}{1 + \alpha}$$

$$v_0 = v_{\text{esc}}\sqrt{\frac{1 + \alpha}{2 + \alpha}}$$

$$v_{\text{esc}}\sqrt{\frac{\alpha}{1 + \alpha}} \le v_0 \le v_{\text{esc}}\sqrt{\frac{1 + \alpha}{2 + \alpha}} \blacktriangleleft$$

Chapter 13, Solution 119.

$$m\mathbf{v}_0 + \int_0^6 \mathbf{F}dt = m\mathbf{v}_6$$

$$0 + \int_0^6 \left[\left(20\sin 2t \right)\mathbf{i} + \left(24\cos 2t \right)\mathbf{j} \right] dt = \frac{3}{32.2}\mathbf{v}_6$$

$$0 + \left[-10\cos 2t\,\mathbf{i} + 12\sin 2t\,\mathbf{j} \right]_0^6 = \frac{3}{32.2}\mathbf{v}_6$$

$$-10\left[\cos 12 - \cos 0 \right]\mathbf{i} + 12\left[\sin 12 - \sin 0 \right]\mathbf{j} = 0.09317\mathbf{v}_6$$

$$1.5615\mathbf{i} - 6.4389\mathbf{j} = 0.09317\mathbf{v}_6$$

$$\mathbf{v}_6 = \left[16.759\mathbf{i} - 69.109\mathbf{j} \right] \text{ft/s}$$

$|v_6| = 71.1$ ft/s ◀

↘ 76.4° ◀

Chapter 13, Solution 120.

$$\int \mathbf{F}dt = m\mathbf{v} - \overset{\mathbf{0}}{\cancel{m\mathbf{v}_0}}$$

$$\int_0^t \big((4t-8)\mathbf{i} - 2\mathbf{j}\big)dt = 4\mathbf{v} = \left(2t^2 - 8t\right)\mathbf{i} - 2t\mathbf{j}$$

$$\mathbf{v} = \left(0.5t^2 - 2t\right)\mathbf{i} - 0.5t\mathbf{j} \text{ m/s}$$

$$|v|^2 = (\text{speed})^2 = \left(0.5t^2 - 2t\right)^2 + (0.5t)^2$$

$$= 0.25t^4 - 2t^3 + 4t^2 + 0.25t^2$$

$$= t^2\left[0.25t^2 - 2t + 4.25\right]$$

$$\frac{d}{dt}\left(\text{speed}^2\right) = 2t\left[0.25t^2 - 2t + 4.25\right] + t^2[0.5t - 2]$$

$$= t^3 - 6t^2 + 8.5t = 0$$

Roots: $t = 0,\ \ v = 0,\ \ t = \dfrac{6 \pm \sqrt{36 - 4(8.5)}}{2}$

$$t = \frac{6 \pm \sqrt{2}}{2} = 2.2929 \text{ s, } 3.7071 \text{ s}(\text{outside interval})$$

At $t = 2.2929$ s, $v = -1.9571\mathbf{i} - 1.1464\mathbf{j}$

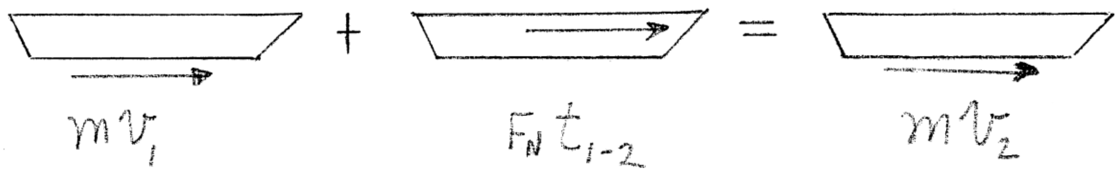
$|v| = 2.27$ m/s, max ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 121.

$$v_1 = 12 \text{ km/h} = 3.33 \text{ m/s} \qquad t_{1-2} = 10 \text{ s}$$

$$v_2 = 18 \text{ km/h} = 5.00 \text{ m/s}$$



$$mv_1 + \text{impulse}_{1-2} = mv_2$$

$$m(333 \text{ m/s}) + F_N(10 \text{ s}) = m(5.00 \text{ m/s})$$

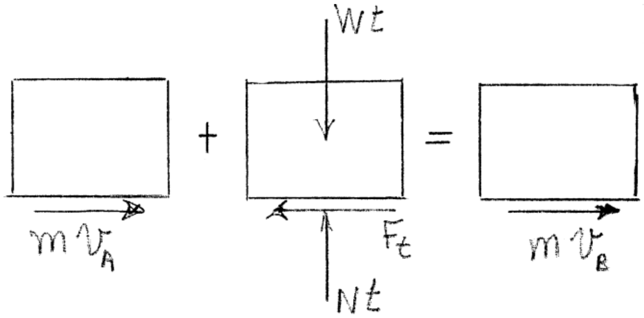
$$F_N = \frac{(440 \text{ kg})(5.00 \text{ m/s} - 3.3333 \text{ m/s})}{10 \text{ s}} = 73.33 \text{ N}$$

$F_N = 73.3$ N ◀

Note: F_N is the net force provided by the sails. The force on the sails is actually greater and includes the force needed to overcome the water resistance of the hull.

Chapter 13, Solution 122.

(*a*) $\theta = 0°$



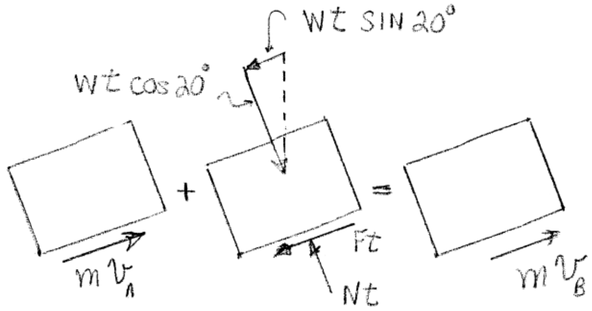
$$Ft = \mu_k Wt = \mu_k mgt$$

$$\cancel{m}v_A - \mu_k \cancel{m}gt = \cancel{m}v_B$$

$$9 - 0.30(9.81)t = 0$$

$t = 3.06$ s ◀

(*b*) $\theta = 20°$



$$Nt = Wt\cos 20° = mgt\cos 20°$$

$$Ft = \mu_k Nt$$

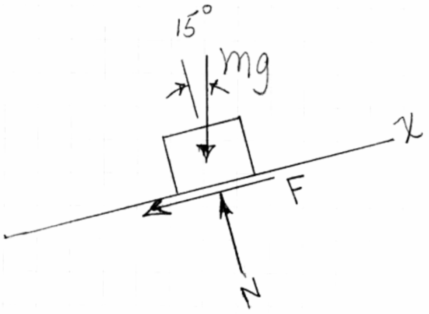
$$\cancel{m}v_A - \mu_k \cancel{m}g\cos 20° t - \cancel{m}gt\sin 20° = 0$$

$$9 - \left[0.65(9.81)\cos 20° + 9.81\sin 20°\right]t = 0$$

$$9 - 9.81(0.9528)t = 0$$

$t = 0.963$ s ◀

Chapter 13, Solution 123.



$$W = 20,000 \text{ lb}$$

$$m = \frac{20,000}{32.2} = 621.118 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Momentum in the x direction

$$x\colon mv_0 - (F + mg\sin 15°)\,t = mv_1$$

$$621.118(108) - (F + mg\sin 15°)6 = (621.118)(36)$$

$$F + mg\sin 15° = 7453.4$$

(*a*) $$F = 7453.4 - 20,000 \sin 15° = 2277 \text{ lb}$$

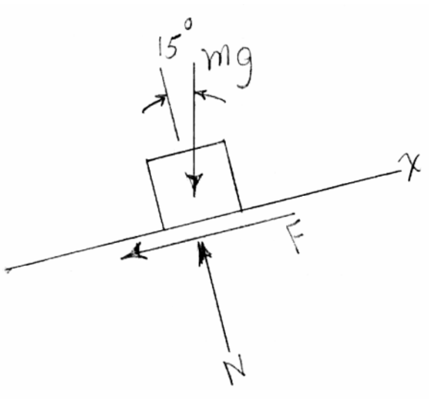
$F = 2280$ lb ◀

(*b*) $mv_0 - (F + mg\ \sin 15°)t = 0$ t = total time

$621.118(108) - 7453.4t = 0;$ $t = 9.00$ s

Additional time = 9 – 6 = 3 s ◀

Chapter 13, Solution 124.



$$W = 20{,}000 \text{ lb}$$

$$m = \frac{20{,}000}{32.2} = 621.118 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Momentum, x: $mv_0 - (F + mg\sin 15°)t = mv_1$

$$621.118(v_0) - (F + mg\sin 15°)(5.5) = 621.118\left(\frac{v_0}{2}\right)$$

$$310.559\, v_0 = (F + mg\sin 15°)(5.5) \qquad (1)$$

Conservation of energy:

$$\frac{1}{2}mv_0^2 - (F + mg\sin 15°)x = \frac{1}{2}mv_1^2$$

$$310.559\, v_0^2 - (F + mg\sin 15°)(540) = \frac{310.559\, v_0^2}{4}$$

$$\frac{3(310.559\, v_0^2)}{4} = (F + mg\sin 15°)(540)$$

Using (1) eliminate $(F + mg\sin 15°)$:

$$310.559\, v_0 = \frac{3(310.559\, v_0^2)}{4}\left(\frac{5.5}{180}\right)$$

(*a*) $$v_0 = \frac{(4)(540)}{3(5.5)} = 130.909 \text{ ft/s} = 89.3 \text{ mi/h} \blacktriangleleft$$

(*b*) $$(F + mg\sin 15°) = \frac{310.559\, v_0}{5.5} = \frac{(310.559)(130.909)}{5.5} = 7391.8$$

$$F = (7391.8 - 20000\sin 15°) = 2215 \text{ lb}$$

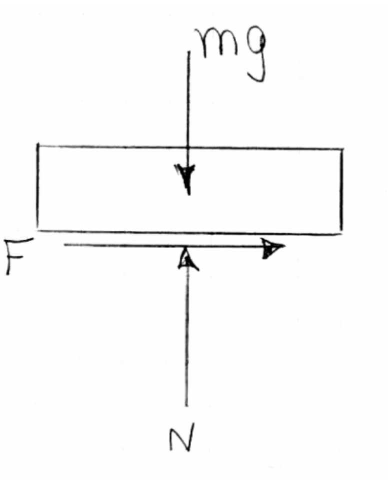
$$F = 2220 \text{ lb} \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 125.

$$v_2 = 100 \text{ km/h} = 27.777 \text{ m/s}$$

$$v_1 = 80 \text{ km/h} = 22.222 \text{ m/s}$$



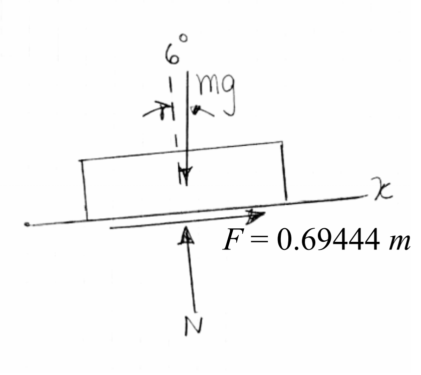
(*a*)

$$\int_0^8 F\,dt = F(8) = m(v_2 - v_1) = m(100 - 80)\left(\frac{1000}{3600}\right)$$

$F = 0.69444\,m$ on the level

on the up grade

$$x: \int_0^{10}(F - mg\sin 6^\circ)\,dt = m(v_{10} - 27.777)$$



$$0.69444\,m(10) - m(9.81)\sin 6^\circ(10) = m(v_{10} - 27.777)$$

(*a*)

$$v_{10} - 27.777 = -3.3098, \quad v_{10} = 24.468 \text{ m/s}$$

$v_{10} = 88.1$ km/h ◀

(*b*)

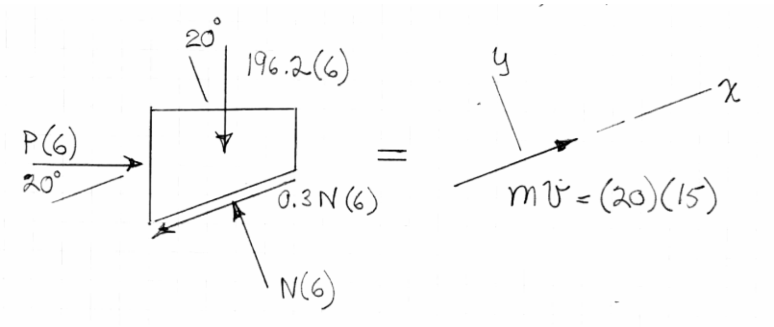
$$\int_0^t (F - mg\sin 6^\circ)\,dt = m(60 - 100)\left(\frac{1000}{3600}\right)$$

$$\left[0.69444\cancel{m} - \cancel{m}(9.81)\sin 6^\circ\right]t = \cancel{m}(-11.111)$$

$t = 33.6$ s ◀

Chapter 13, Solution 126.

$$W = (20)(9.81) = 196.2 \text{ N}$$



Impulse diagonally (assume sliding)

$$x: \left[P\cos 20° - 196.2\sin 20° - 0.3\text{ N}\right](6) = 20(15)$$

$$y: \left[\text{N} - 196.2\cos 20° - P\sin 20°\right](6) = 0$$

$$P\cos 20° - 196.2\sin 20° - 0.3\left[196.2\cos 20° + P\sin 20°\right] = 20\left(\frac{15}{6}\right) = 50$$

$$P\left(\cos 20° - 0.3\sin 20°\right) = 50 + 67.104 + 55.310$$

$$P = 205.97 \text{ N} \qquad P = 206 \text{ N} \blacktriangleleft$$

Check μ_s

Static value $\quad 0.4\text{ N} = 0.4\left(196.2\cos 20° + P\sin 20°\right)$

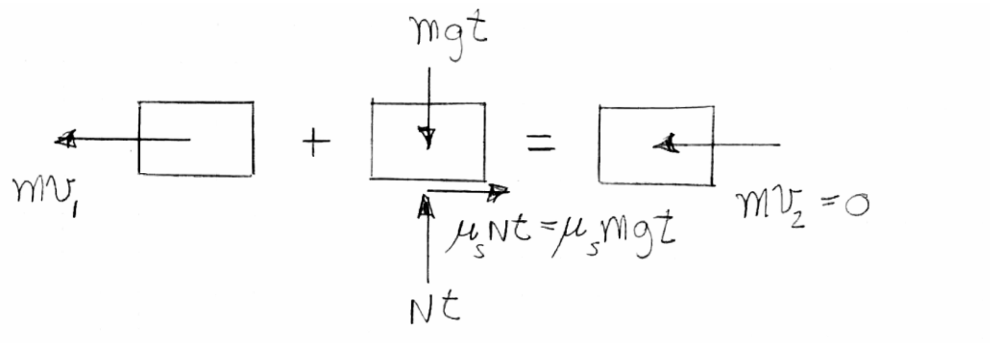
$$P\cos 20° - 196.2\sin 20° - 0.4\text{ N} = 0$$

$$P\left(\cos 20° - 0.4\sin 20°\right) = 196.2\sin 20° + 78.48\cos 20° = 140.85$$

$$P_{\text{static}} = 175.4 \text{ N} < 206 \text{ N}$$

Chapter 13, Solution 127.

Use impulse momentum $v_1 = 55\,\text{mi/h} = 80.667\ \text{ft/s}$



x-Direction ←

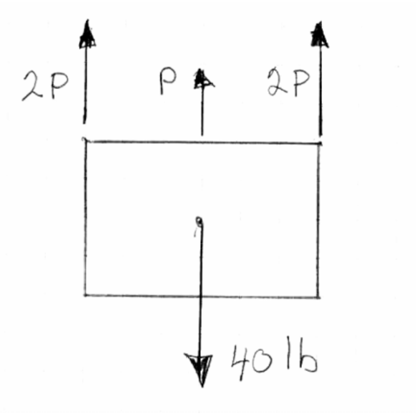
$$\cancel{m} v_1 - \mu_s \cancel{m} gt = 0$$

$$t = \frac{v_1}{\mu_s g} = \frac{80.667\ \text{ft/s}}{(0.4)\left(32.2\ \text{ft/s}^2\right)}$$

$t = 6.26\ \text{s}$ ◀

Since this is the shortest time the load can be brought to rest and the load does not slide it is also the shortest time the rig can be brought to rest.

Chapter 13, Solution 128.



$+\!\uparrow \qquad \int_{t=0}^{t=4} F\,dt = m(v_2 - v_1)$

$$\int_0^4 (5P - 40)\,dt = (40/32.2)(2 - (-3))$$

$$20P - 40(4) = (40/32.2)(5)$$

(*a*) $P = (6.2112 + 160)/20$

$P = 8.31$ lb ◀

(*b*) $\int_0^t (5P - 40)\,dt = (40/32.2)(0 - (-3))$

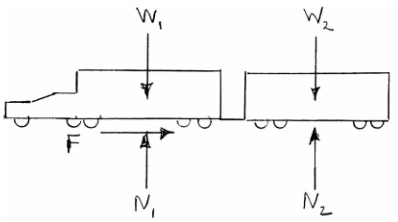
$$5P\mathrm{t} - 40t = (40/32.2)(3)$$

$t = 2.4$ s ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 129.

(*a*) Combined



$v = 90\,\text{km/h} = 25\ \text{m/s}$

$W_1 = (6500)(9.81) = 63765\ \text{N}; \quad W_2 = (3600)(9.81) = 35316\ \text{N}$

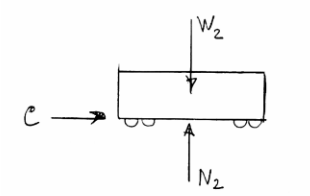
$N_1 = W_1; \quad N_2 = W_2 \qquad F = 0.75N_1$

$\overset{+}{\underset{\otimes}{\longleftarrow}}$ Impulse = $0 - mv_0$

$$-0.75N_1 t = -10{,}100\ \text{kg}(25\ \text{m/s})$$

$$t = \frac{10{,}100(25)}{(0.75)(63765)} = 5.2798\ \text{s} \quad t = 5.28\ \text{s} \blacktriangleleft$$

(*b*) Second trailer alone



$\overset{+}{\underset{\otimes}{\longleftarrow}}$ Impulse $= -Ct = m_2 v$

$$-C(5.2798) = -3600\ \text{kg}\ (25\ \text{m/s})$$

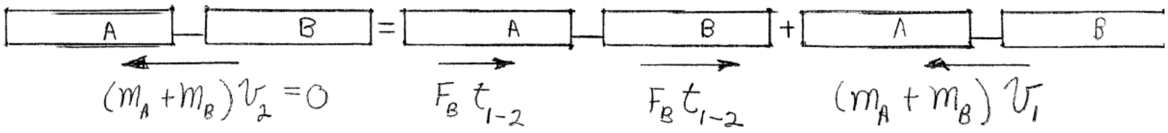
$$C = 17046\ \text{N}$$

$C = 17.05\ \text{kN} \blacktriangleleft$

Compression

Chapter 13, Solution 130.

(*a*) *Entire train* $v_1 = 72\text{ km/h} = 20\text{ m/s}$



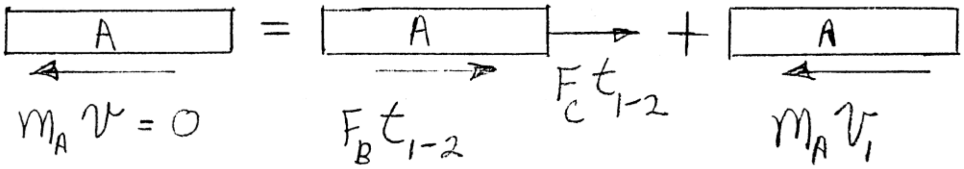
$$m_A + m_B = 18 + 13 = 31\text{ Mg} = 31\times 10^3\text{ kg}$$

$$\overset{+}{\leftarrow}\ 0 = -(19000\text{ N} + 19000\text{ N})t_{1-2} + \left(31\times 10^3\text{ kg}\right)(20\text{ m/s})$$

$$t_{1-2} = \frac{\left(31\times 10^3\text{ kg}\right)(20\text{ m/s})}{38000\text{ N}} = 16.3158\text{ s}$$

$t_{1-2} = 16.32\text{ s}$ ◀

(*b*) *Car A* $m_A = 18\text{ Mg} = 18\times 10^3\text{ kg};\quad t_{1-2} = 16.32\text{ s}$



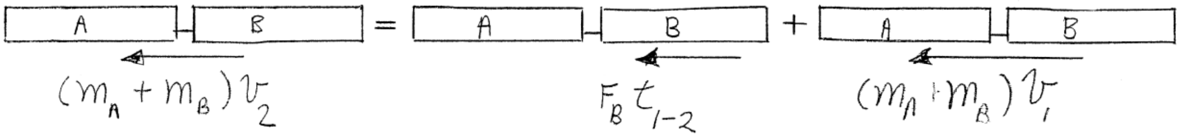
$$\overset{+}{\leftarrow}\ 0 = \left[(19000\text{ N}) + F_C\right]\left[16.32\text{ s}\right] + (18000\text{ kg})(20\text{ m/s})$$

$$F_C = 3058.8\text{ N}$$

$F_C = 3.06\text{ kN T}$ ◀

Chapter 13, Solution 131.

(*a*) *Entire train* $v_1 = 72 \text{ km/h} = 20 \text{ m/s}$



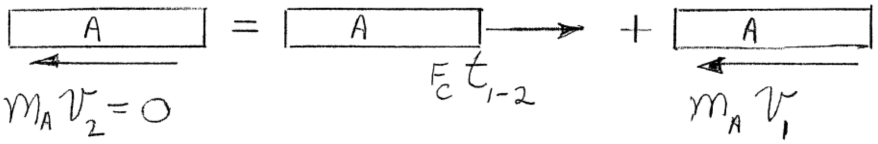
$$m_A + m_B = 18 + 13 = 31 \text{ Mg} = 31000 \text{ kg}$$

$$\overset{+}{\leftarrow}\ 0 = -(19000 \text{ N})t_{1-2} + (31000 \text{ kg})(20 \text{ m/s})$$

$$t_{1-2} = 32.63 \text{ s}$$

$t_{1-2} = 32.6 \text{ s}$ ◀

(*b*) *Car A*

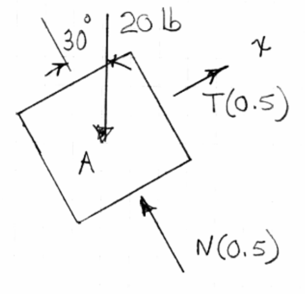


$$\overset{+}{\leftarrow}\ 0 = -F_C t_{1-2} + m_A v_1; \qquad t_{1-2} = 32.63 \text{ s}$$

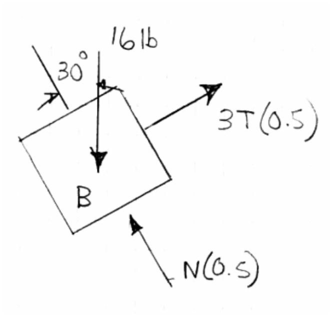
$$F_C = \frac{(18000 \text{ kg})(20 \text{ m/s})}{(32.63 \text{ s})} = 11033 \text{ N}$$

$F_C = 11.03 \text{ kN T}$ ◀

Chapter 13, Solution 132.



Impulse diagrams



Constraint: $v_A \swarrow\; = 3 v_B \nearrow$

$A: +\swarrow x \qquad 20(0.5)\sin 30° - T(0.5) = \frac{20}{32.2} v_A$

$B: +\nearrow x \qquad 3T(0.5) - 16(0.5)\sin 30° = \frac{16}{32.2} v_B = \frac{16\, v_A}{32.2(3)}$

Substituting for $T(0.5)$ from the equation for A into the equation for B

From A:

$$T(0.5) = 5 - 0.62112\, v_A$$

$$15 - 1.8634\, v_A - 4 = \frac{0.4969\, v_A}{3}$$

$$2.029\, v_A = 11$$

$$v_A = 5.4214$$

(*a*) $v_A = 5.42$ ft/s ↙ 30° ◀

$$T(0.5) = 5 - 0.62112(5.4214)$$

$$T = 3.2653 \text{ lb}$$

(*b*) $T = 3.27$ lb ◀

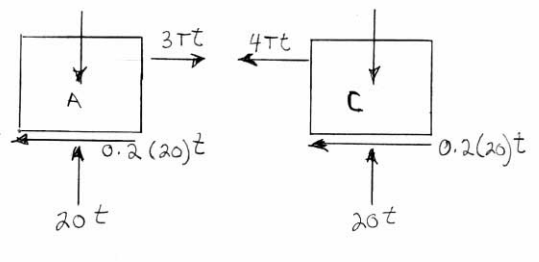
Chapter 13, Solution 133.

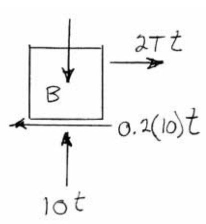
$$l \cong 3\left(x_C - x_A\right) + \left(x_C - x_B\right) + \left(d - x_B\right)$$

Constraint: $4v_C - 2v_B - 3v_A = 0$ (1)

For v_A, v_B and v_C $+\longrightarrow$

At $t = 0$, $4\left(15\,\text{ft/s}\right) - 2v_B - 3\left(9\,\text{ft/s}\right) = 0$, $v_B = 16.5$ ft/s





(2) $3Tt - 4t = \dfrac{20}{32.2}\left(v_A - 9\right)$ (3) $-4Tt - 4t = \dfrac{20}{32.2}\left(v_C - 15\right)$ (4) $2Tt - 2t = \dfrac{10}{32.2}\left(v_B - 16.5\right)$

Given $t = 0.5$ s $\Rightarrow$ 4 Equations in T, v_A, v_B, v_C

$$\begin{aligned} -3v_A - 2v_B + 4v_C &= 0 && (1)\\ 1.5T - 0.6211\,v_A &= -3.5901 && (2)\\ -2T - 0.6211\,v_C &= -7.3168 && (3)\\ T - 0.31056\,v_B &= -4.1242 && (4) \end{aligned}$$

From the solution of the above equations.

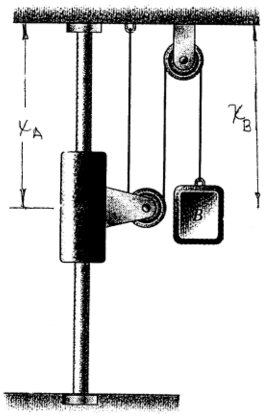
(*a*) $v_A = 6.07$ ft/s $\longrightarrow$ ◀

$v_B = 13.7$ ft/s $\longrightarrow$ ◀

$v_C = 11.4$ ft/s $\longrightarrow$ ◀

(*b*) $T = 0.1212$ lb ◀

Chapter 13, Solution 134.



Kinematics

Length of cable is constant.

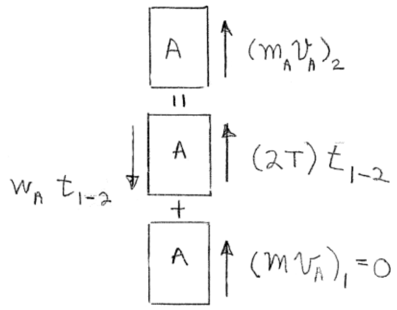
$$L = 2X_A + X_B$$

$$\frac{dL}{dt} = 2v_A + v_B = 0$$

$$v_B = -2v_A$$

$$(v_A)_2 = 0.6 \text{ m/s}$$

Collar A



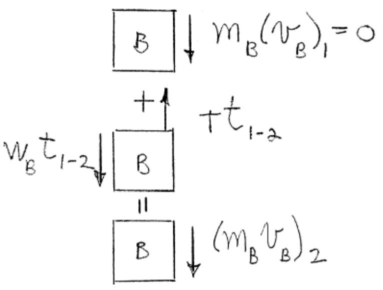
$$m_A = 15 \text{ kg}$$

$$(m_A v_A)_1 + (2T)(t_{1-2}) - W_A t_{1-2} = m(v_A)_2$$

$$0 + \left[2T - (15 \times 9.81)\right]t_{1-2} = (15)(0.6)$$

$$(T - 73.575)t_{1-2} = 4.5 \qquad (1)$$

Collar B



$$m_B = 10 \text{ kg}$$

$$(v_B)_2 = 2(v_A)_2 = 1.2 \text{ m/s}$$

$$+\!\downarrow\; (m_B v_A)_1 - T(t_{1-2}) + W_B(t_{1-2}) = (m_B v_B)_2$$

$$0 + \left[(10 \times 9.81) - T\right](t_{1-2}) = 10(1.2) \qquad (2)$$

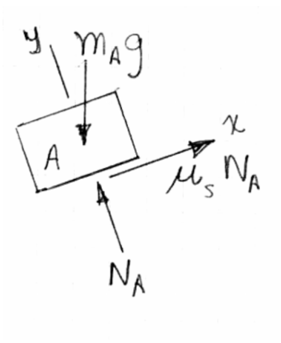
Add Equation (1) and Equation (2) (eliminating *T*)

$$(98.1 - 73.575)t_{1-2} = 4.5 + 12$$

$$t_{1-2} = \frac{16.5}{24.52} = 0.673 \text{ s} \qquad t = 0.673 \text{ s} \blacktriangleleft$$

Chapter 13, Solution 135.

Lets find out if they slide – assume they don't slide and find the required angle for impending motion



↖ $\sum F_y = 0; \Rightarrow N_A - m_A g \cos\theta_i = 0; \Rightarrow N_A = m_A g \cos\theta_i$

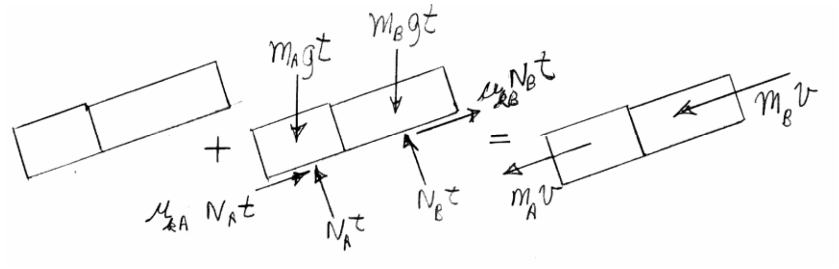
↗ $\sum F_x = 0; \Rightarrow \mu_s N_A - m_A g \sin\theta_i = 0$

$$\mu_s \cancel{m}_A \cancel{g} \cos\theta_i = \cancel{m}_A \cancel{g} \sin\theta_i$$

$$\tan\theta_i = \mu_s = 0.3$$

so $\theta_i = 16.7°$ so they slide

Assume they slide at the same velocity (remain in contact)
impulse – momentum



x - dir +↙

$$0 + (m_A g + m_B g) t \sin\theta - (\mu_{kA} N_A + \mu_{kB} N_B) t = (m_A + m_B) v$$

(a) Solve for v

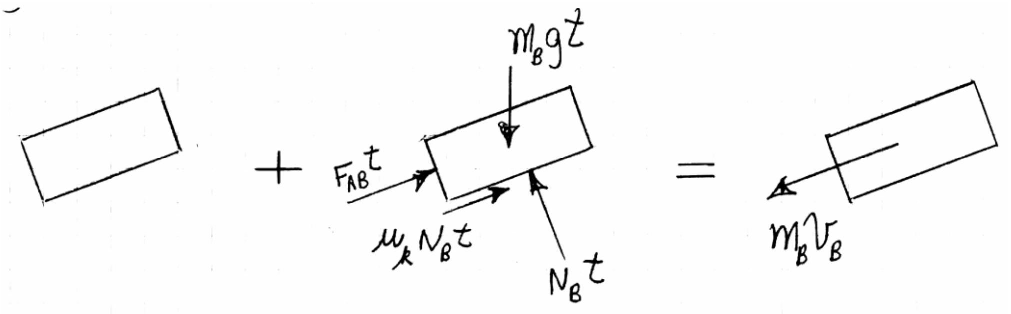
$$v = \frac{(m_A g + m_B g) t \sin\theta - [\mu_{kA}\, m_A g \cos\theta + \mu_{kB}\, m_B g \cos\theta] t}{m_A + m_B}$$

$$= \frac{(6+9)(9.81)(3)\sin 20° - \left[(0.25)(6) + 0.15(9)9.81\cos 20°\right](3)}{6+9}$$

$\overline{v} = 4.811$ m/s

$\overline{v}_A = \overline{v}_B = 4.81$ m/s ↙ ◀

(*b*) Just look at *AB*



$$x+\swarrow \quad 0 + m_B\, gt\, \sin\theta - F_{AB}t - \mu_k\, N_B t = m_B v_B$$

So $$F_{AB} = \frac{m_B\, gt\, \sin\theta - \mu_k\left(m_B\, g\, \cos\theta\right)t - m_B v_B}{t}$$

$$= \frac{(9)(9.81)(3)\sin 20° - 0.15(9)(9.81)\cos 20°(3) - (4.81)(9)}{3}$$

$$= 3.319 \text{ N}$$

$\overline{F}_{AB} = 3.32$ N ◀

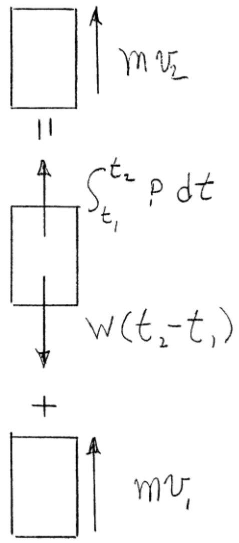
Since this is positive our assumption that the blocks stay in contact is correct

Chapter 13, Solution 136.

The block does not move until $P = 2\text{ kg} \times 9.81\text{ m/s}^2 = 19.62\text{ N}$

From $t = 0$ to $t = 2\text{ s}$ $\quad P = 20t$

Thus, the block starts to move when $t = \dfrac{19.62}{20} = 0.981\text{ s}$



(*a*) For $\quad 0 < t < 2\text{ s}$

$$P = 20t$$

$$t_1 = 0.981\text{ s} \qquad t_2 = 2\text{ s},\ v_1 = 0$$

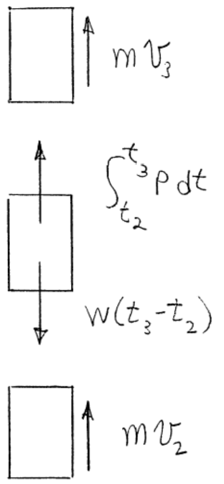
$$mv_1 + \int_{t_1}^{t_2} P dt - W(t_2 - t_1) = mv_2$$

$$0 + \int_{0.981}^{2} 20t\,dt - (2 \times 9.81)(2 - 0.981) = 2v_2$$

$$v_2 = \frac{1}{2\text{ kg}}\left[\frac{20\text{ N}}{2\text{ s}}\left[(2\text{ s})^2 - (0.981\text{ s})^2\right] - (19.62\text{ N})(2\text{ s} - 0.981\text{ s})\right]$$

$$v_2 = 5.1918\text{ m/s}$$

$$v_2 = 5.19\text{ m/s}\uparrow \blacktriangleleft$$



(*b*) From $t = 2\text{ s}$ to $t = 3\text{ s}$

$$v_2 = 5.19\text{ m/s}, \quad \text{from } (a)$$

$$P = 40\text{ N} \qquad 2\text{ s} \le t \le 3\text{ s}$$

$$t_2 = 2\text{ s} \qquad t_3 = 3\text{ s}$$

$$mv_2 + \int_{t_2}^{t_3} P dt - W(t_3 - t_2) = mv_3$$

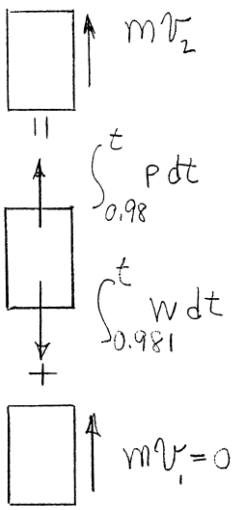
$$2(5.1918) + \int_2^3 40dt - (19.62)(3 - 2) = 2v_3$$

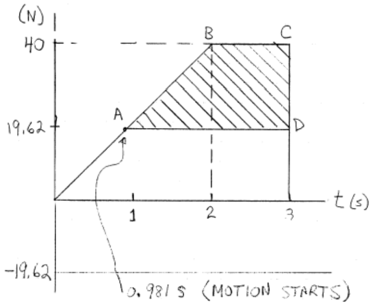
$$v_3 = (5.1918\text{ m/s}) + \frac{1}{(2\text{ kg})}\left[(20.38\text{ N})(1\text{ s})\right]$$

$$v_3 = 5.1918 + 10.19 = 15.3818\text{ m/s}$$

$$v_3 = 15.38\text{ m/s}\uparrow \blacktriangleleft$$

Chapter 13, Solution 137.





(*a*) Determine time at which collar starts to move.

$$P = 20t, 0 < t < 2\text{ s}$$

Collar moves when

$$P = 2\text{ kg}\times 9.81\text{ m/s}^2, \qquad \text{or} \qquad t = \frac{P}{20} = \frac{19.62}{20} = 0.981\text{ s}$$

$$mv_1 + \int_{0.981}^{t} Pdt - \int_{0.981}^{t} Wdt = mv_2$$

For $\quad t < 2\text{ s} \quad P = 20t(\text{N})$

$2\text{ s} < t < 3\text{ s} \quad P = 40\text{ N}$

$t > 3\text{ s} \quad P = 0$

For $\quad t < 3\text{ s} \quad W = 2\text{ kg}\times 9.81\text{ m/s}^2$

The maximum velocity occurs when the total impulse is maximum.

$$\text{area}_{ABCD} = \text{maximum impulse}$$

$$= \frac{1}{2}(20.38\text{ N})(1.019\text{ s}) + (20.38\text{ N})(1\text{ s})$$

$$\text{area}_{ABCD} = 30.76\text{ N}\cdot\text{s}$$

$$0 + 30.76\text{ N}\cdot\text{s} = (2\text{ kg})v_{\max}$$

$$v_{\max} = 15.38\text{ m/s}\uparrow \blacktriangleleft$$

(*b*) Velocity is zero when total impulse is zero at $t + \Delta t$.

For $\quad 0.981\text{ s} < t < 3\text{ s}, \ \text{impulse} = -19.62\Delta t(\text{N}\cdot\text{s})$

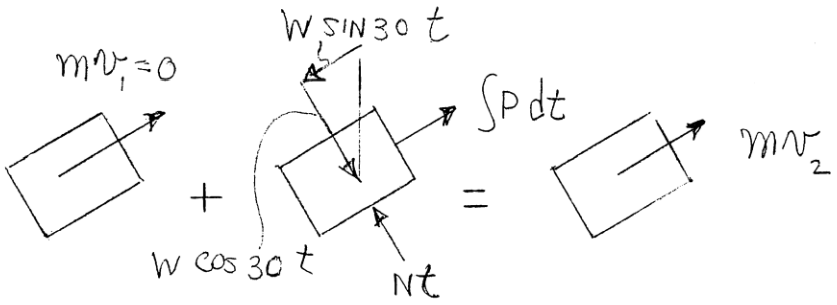
Thus, $\quad \text{total impulse} = 0 = 30.76 - 19.62\Delta t$

$$\Delta t = 1.57\text{ s}$$

Time to zero velocity– $t = 3\text{ s} + 1.57\text{ s} = 4.57\text{ s}$ ◀

Chapter 13, Solution 138.

(*a*)



$$P = 20 - 4t$$

$$\underset{0}{\cancel{mv_1}} - W\sin 30t + \int_0^t P\,dt = mv_2$$

$$-12\sin 30°(5) + \int_0^5 (20 - 4t)\,dt = \frac{12}{32.2}v_2$$

$$-30 + \left[20t - 2t^2\right]_0^5 = 0.37267v_2$$

$$v_2 = 53.7 \text{ ft/s} \blacktriangleleft$$

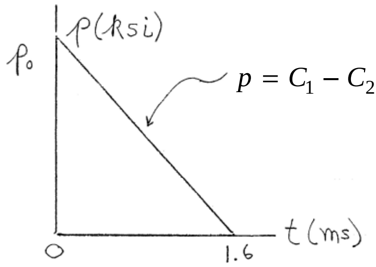
(*b*) After $t = 5$ s, $P = 0$ $v_5 = 53.7$ ft/s, t' is time after 5 s

$$mv_5 - 12\sin 30t' = 0$$

$$\frac{12}{32.2}(53.7) = 6t' \Rightarrow t' = 3.34 \text{ s}; \qquad t = 5 + t'$$

$$t = 8.34 \text{ s} \blacktriangleleft$$

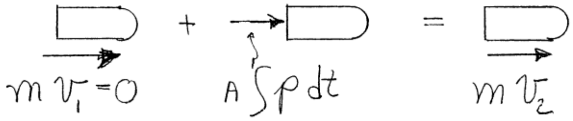
Chapter 13, Solution 139.



$$p = C_1 - C_2 t, \qquad \text{where} \qquad C_1 = p_0, \qquad C_2 = \frac{p_0}{1.6\times 10^{-3}}$$

$$m = \frac{\left(\dfrac{0.70\text{ oz}}{\frac{16\text{ oz}}{1\text{ lb}}}\right)}{g} = \frac{0.04375}{g}$$

$$A = \frac{\pi}{4}(0.4)^2 = 0.12566\text{ m}^2$$



$$0.12566\int_0^{1.6\times10^{-3}} (C_1 - C_2 t)\,dt = \frac{0.04375(2100\text{ ft/s})}{(32.2\text{ ft/s}^2)} = 2.85326$$

$$\left(C_1 t - \frac{1}{2}C_2 t^2\right)\Bigg|_0^{1.6\times10^{-3}} = 22.706$$

$$1.6\times10^{-3}\text{ s}(p_0) - \frac{(1.6\times10^{-3}\text{ s})^2}{2}\frac{p_0}{1.6\times10^{-3}\text{ s}} = 22.706$$

$$\frac{(1.6\times10^{-3}\text{ s})\,p_0}{2} = 22.706$$

$$p_0 = 28.383\text{ lb/in}^2$$

$p_0 = 28.4$ ksi ◀

Chapter 13, Solution 140.

$$m = 2\text{ oz}\left(1\text{ lb}/16\text{ oz}\right)\left(\frac{1}{32.2\text{ ft/s}^2}\right) = 0.003882\text{ lb}\cdot\text{s}^2/\text{ft}$$

Conservation of energy (before impact)

$$\frac{1}{2}\cancel{m}\,v_1^2 + \cancel{m}gh_1 = \frac{1}{2}\cancel{m}\left(v_1^2 + v_{Ay}^2\right)$$

$$\frac{1}{2}\cancel{m}\,(54)^2 + \cancel{m}\,(32.2)\,(4.5) = \frac{1}{2}\cancel{m}\left(54^2 + v_{Ay}^2\right)$$

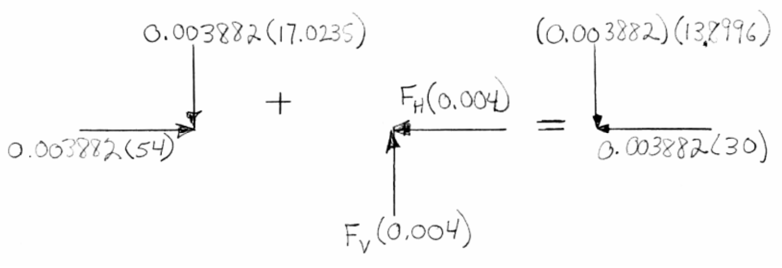
$$v_{Ay} = 17.0235\text{ ft/s (Just before impact)}$$

Conservation of energy (after impact)

$$\frac{1}{2}m\left(v_{Ay}'^2 + v_2^2\right) = \frac{1}{2}m\left(v_2\right)^2 + mg\,h_2$$

$$\frac{1}{2}\cancel{m}\left[v_{Ay}'^2 + (30)^2\right] = \frac{1}{2}\cancel{m}(30)^2 + \cancel{m}(32.2)(3)$$

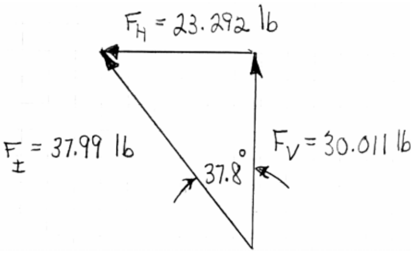
$$v_{Ay}' = 13.8996\text{ ft/s (Just after impact)}$$



$$\xrightarrow{+} x: 0.003882(54) - F_H(0.004) = 0.003882(30),\ F_H = 23.292\text{ lb}$$

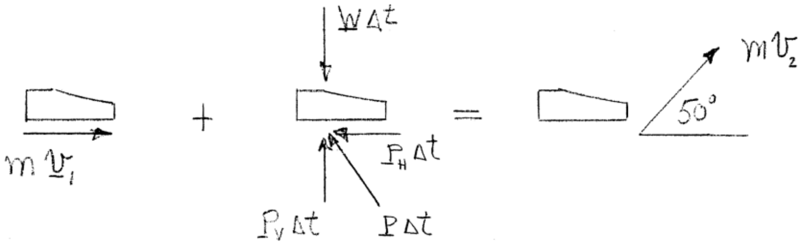
$$+\!\uparrow y: -0.003882(17.0235) + F_v(0.004) = 0.003882(13.8996),\ F_v = 30.011\text{ lb}$$

$$F_I = \text{impulsive force}$$


$F_H = 23.292$ lb
$F_I = 37.99$ lb
$F_V = 30.011$ lb
37.8°

$F_I = 38.0$ lb ⦤ 52.2° ◀

Chapter 13, Solution 141.


$W\Delta t$
$m\mathbf{v}_2$
$50°$
$m\mathbf{v}_1$
$P_H \Delta t$
$P_V \Delta t$
$P \Delta t$

$$v_1 = 30 \text{ ft/s} \qquad v_2 = 36 \text{ ft/s} \qquad \Delta t = 0.18 \text{ s}$$

$$mv_1 + (P - W)\Delta t = mv_2$$

Vertical components $\uparrow +$

$$0 + (P_V - W)(0.18) = \frac{W}{g}(36 \sin 50°)$$

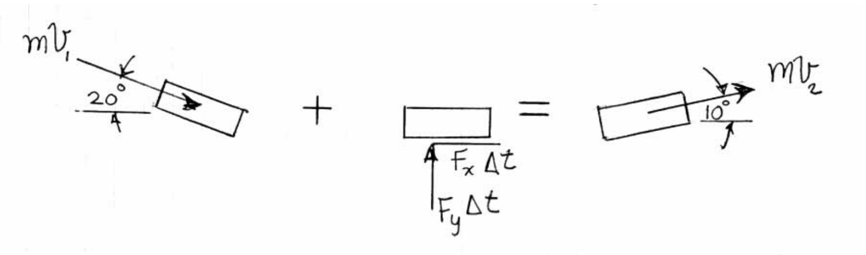
$$P_V - W = 4.758\,W$$

$$P_V = 5.76\,W \blacktriangleleft$$

Chapter 13, Solution 142.

Use impulse–momentum for bullet

Knowns: $m = 0.028$ kg, $v_1 = 650$ m/s, $v_2 = 500$ m/s,



x-dir →

$$mv_1 \cos 20° - F_x \Delta t = mv_2 \cos 10°$$

So,

$$F_x \Delta t = mv_1 \cos 20° - mv_2 \cos 10°$$

$$= 0.028\,(650)\,\cos 20° - 0.028\,(500)\,\cos 10° = 3.3151\ \text{N}\cdot\text{s}$$

y-dir ↑

$$-mv_1 \sin 20° + F_y \Delta t = mv_2 \sin 10°$$

So,

$$F_y \Delta t = mv_2 \sin 10° + mv_1 \sin 20°$$

$$= 0.028\,(500)\,\sin 10° + 0.028\,(650)\,\sin 20° = 8.6558\ \text{N}\cdot\text{s}$$

We need Δt. The average velocity is 600 m/s

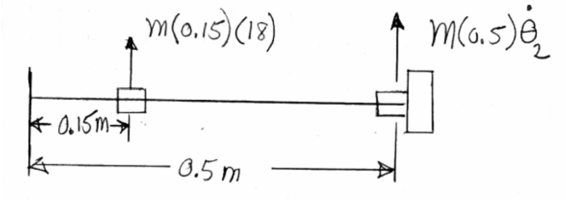
$$\Delta x = v_{\text{ave}}\ \Delta t;\ \Delta t = \frac{\Delta x}{v_{\text{ave}}} = \frac{0.05\ \text{m}}{600\ \text{m/s}} = 83.33 \times 10^{-6}\ \text{s}$$

So

$$\left.\begin{aligned} F_x &= \frac{3.3151}{83.33 \times 10^{-6}} = 39.78\ \text{kN} \\ \text{F}_y &= \frac{8.6558}{83.33 \times 10^{-6}} = 103.87\ \text{kN} \end{aligned}\right\}\ \bar{F} = 111.2\ \text{kN}\ \ 69.0°$$ ◀

Chapter 13, Solution 143.

Conservation of angular momentum



$$m(0.15)\,(18)\,(0.15) = m(0.5)\dot{\theta}_2\,(0.5)$$

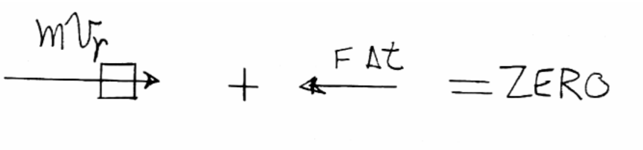
$$\dot{\theta}_2 = 1.62 \text{ rad/s}$$

Conservation of energy:

$$\frac{1}{2}\,m\left[0.15(18)\right]^2 = \frac{1}{2}\,m\left[\left(0.5(1.62)\right)^2 + \dot{R}^2\right]$$

$$\dot{R} = 2.5756 \text{ m/s}$$

Motion relative to the rod:

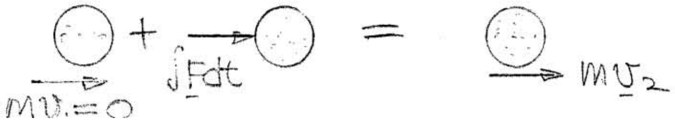


$$(1.5)\,(2.5756) - F\,\Delta t = 0$$

$$F\,\Delta t = 3.86 \text{ N}\cdot\text{s} \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr., Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 144.



$$v_2 = 38 \text{ m/s}$$

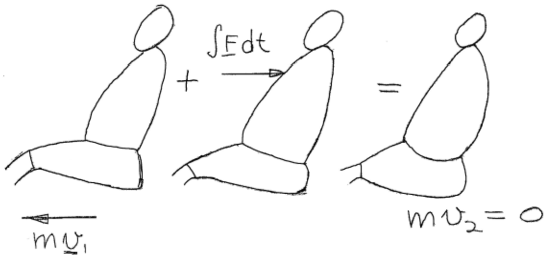
$$mv_1 + \int_0^{t_0} F dt = mv_2$$

$$0 + \int_0^{0.5\times10^{-3}} F_m \sin\frac{\pi t}{0.5\times10^{-3}} dt = (0.045 \text{ kg})(38 \text{ m/s})$$

$$F_m = 5.37 \text{ kN} \blacktriangleleft$$

Chapter 13, Solution 145.

(*a*) Force on the belt is opposite to the direction shown.



$$v_1 = 72 \text{ km/h} = 20 \text{ m/s}, \qquad m = 100 \text{ kg}$$

$$mv_1 - \int F dt = mv_2 \qquad \int F dt = F_{\text{ave}} \Delta t$$

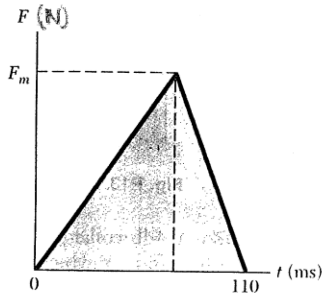
$$(100 \text{ kg})(20 \text{ m/s}) - F_{\text{ave}}(0.110 \text{ s}) = 0 \qquad \Delta t = 0.110 \text{ s}$$

$$F_{\text{ave}} = \frac{(100)(20)}{(0.110)} = 18182 \text{ N}$$

$F_{\text{ave}} = 18.18$ kN ◀

(*b*) Impulse = area under $F - t$ diagram

$$= \frac{1}{2} F_m (0.110 \text{ s})$$



From (*a*) $\quad$ Impulse $= F_{\text{ave}} \Delta t$

$$= (18182 \text{ N})(0.110 \text{ s})$$

$$\frac{1}{2} F_m (0.110) = 18182(0.110)$$

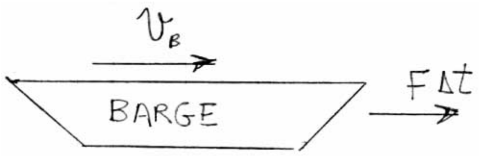
$F_m = 36.4$ kN ◀

Chapter 13, Solution 146.

$$m_B = (100)(2000)\left(\frac{1}{32.2}\right) = 6211.18 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_T = (120)(2000)\left(\frac{1}{32.2}\right) = 7453.42 \text{ lb}\cdot\text{s}^2/\text{ft}$$

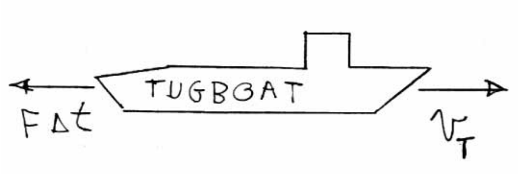
$$0 + F\,\Delta t = m_B v_B = (6211.18)\, v_B$$



$$m_T(6) - F\,\Delta t = m_T v_T$$

$$(7453.42)(6) - F\,\Delta t = (7453.42)\, v_T$$

constraint: $v_{T/B} = v_T - v_B = 5.4$ ft/s



Solving; $7453.42(6) = (6211.18)[v_T - 5.4] + 7453.42\, v_T$

$$(13664.6)\, v_T = 78260.9$$

$$v_T = 5.7273 \text{ ft/s}$$

(*a*) $v_T = 5.73$ ft/s → ◀

$$v_B = 5.7273 \text{ ft/s} - 5.4 \text{ ft/s} = 0.3273 \text{ ft/s}$$

$$F\,\Delta t = m_B v_B = (6211.18)(0.3273) = 2032 \text{ lb}\cdot\text{s}$$

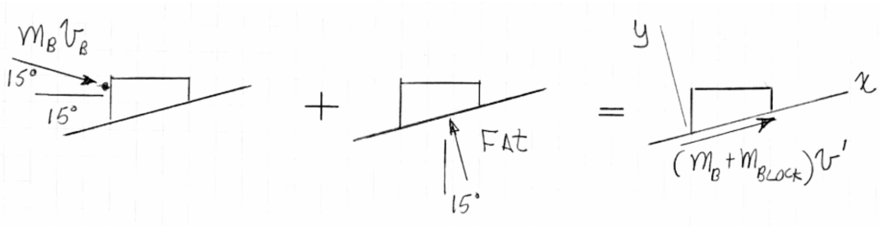
(*b*) $F\,\Delta t = 2030$ lb·s ◀

Chapter 13, Solution 147.

$$W_B = \frac{1}{16}\text{ lb} = 0.0625\text{ lb};\quad m_B = \left(\frac{1}{16}\right)\left(\frac{1}{32.2}\right) = 0.001941\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$W_{\text{block}} = 8\text{ lb};\quad m_B = 0.248447\text{ lb}\cdot\text{s}^2/\text{ft}$$

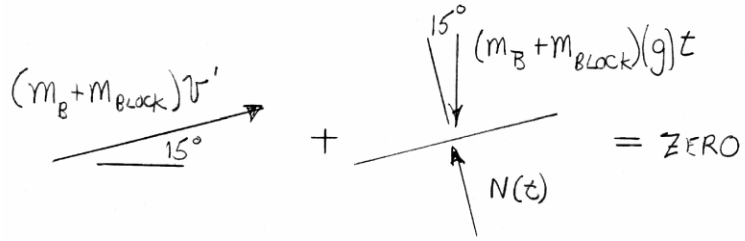
Initial impact (Bullet + Block)



$$\xrightarrow{+} x : m_B v_0 \cos 30° + 0 = \left(m_B + m_{\text{block}}\right)v' \qquad (1)$$

$$+\!\uparrow\; y : -m_B v_0 \sin 30° + F\Delta t = 0 \qquad (2)$$

After impact



$$x : \left(m_B + m_{\text{block}}\right) v' - \left(m_B + m_{\text{block}}\right)(32.2)\,(1.2)\,\sin 15° = 0$$

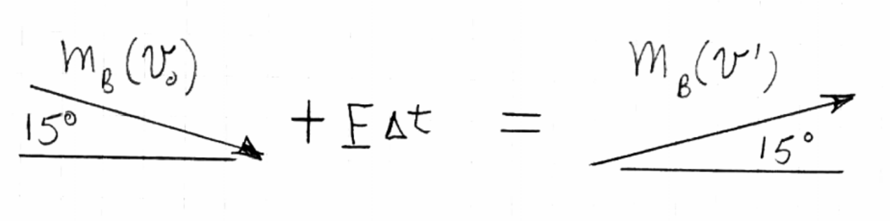
$$v' - gt\sin 15° = 10.001\text{ ft/s}$$

From (1)

$$v_0 = \frac{\left[\dfrac{\left(W_B + W_{\text{block}}\right)}{g}\right]v'}{\left[\dfrac{W_B}{g}\right]\cos 30°} = \frac{(8.0625)\,(10.001)}{0.0625\cos 30°}$$

(*a*) $v_0 = 1489.7$ ft/s $\qquad v_0 = 1490$ ft/s ◀

(*b*) Bullet alone



$$m_B\, v_0 = (0.001941)\,(1489.7) = 2.8915; \quad m_B\, v' = (0.001941)\,(10.001) = 0.01941$$

$x \ \xrightarrow{+}$: $\quad m_B\, v_0 \cos 15° + F_x\,\Delta t = m_B\, v' \cos 15°$

y $+\!\uparrow$: $\quad -m_B\, v_0 \sin 15° + F_y\,\Delta t = m_B\, v' \sin 15°$

Solve: $\quad F_x\,\Delta t = -2.7742$

$F_y\,\Delta t = 0.7534 \quad |F\,\Delta t| = 2.87 \text{ lb}\cdot\text{s}$ ◀

Chapter 13, Solution 148.

$$\mathbf{v}_A = \mathbf{v}_{A/B} + \mathbf{v}_B : \mathbf{v}_A = \frac{\overset{\searrow}{30^\circ}}{3.59\text{ m/s}} + \xleftarrow{\quad} \mathbf{v}_B$$

$$\xrightarrow{+} m_A v_{Ax} + m_B v_{Bx} = 0 \qquad 2\left[3.59\cos 30^\circ - v_B\right] + 10\left(-v_B\right) = 0$$

$$v_B = 0.518\text{ m/s Just before impact } \longleftarrow$$

After impact, $v_A = v_B = 0$

(*a*) $\therefore F\,\Delta t = -m_A v_A = -2\left[\underset{30^\circ}{\overset{3.59}{\searrow}} + \overset{0.5182}{\longleftarrow}\right] = \underset{5.1817}{\longleftarrow}\ \overset{3.59}{\uparrow}$

$$F\,\Delta t = 6.30\text{ N}\cdot\text{S} \quad \underset{34.7^\circ}{\nwarrow} \quad ◀$$

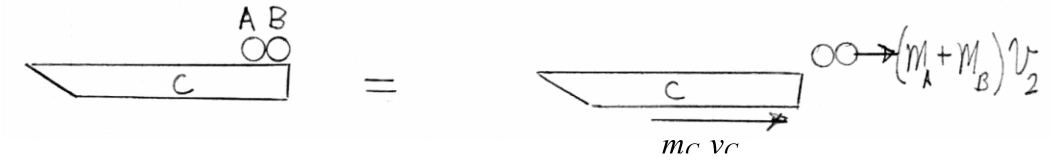
(*b*) $\text{Loss} = T,\ \text{just before impact} = \frac{1}{2}\left(2\,v_A^2\right) + \frac{1}{2}\left(10\,v_B^2\right)$

$$T = \left[\frac{6.3038}{2}\right]^2 + 5\left(0.51817\right)^2 = 11.28\text{ J} ◀$$

Chapter 13, Solution 149.

$$m_A = 75 \text{ kg}, \quad m_B = 50 \text{ kg}, \quad m_C = 200 \text{ kg (Boat)}$$

(*a*) Swimmers dive simultaneously



$$0 = m_C v_C + (m_A + m_B) v_2 \qquad (1)$$

Relative velocity of swimmers with respect to the boat is 3 m/s

$v_2 - v_C = 3 \text{ m/s} \Rightarrow v_2 = v_C + 3$

Substitute into (1)

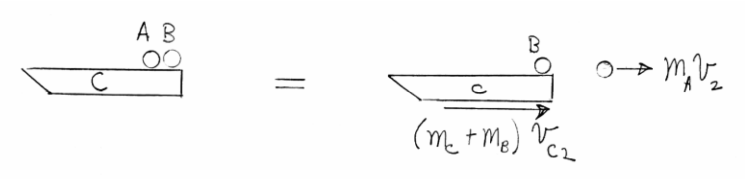
$$0 = m_C v_C + (m_A + m_B)(v_C + 3)$$

Solve

$$v_C = \frac{-3(m_A + m_B)}{m_A + m_B + m_C} = \frac{-3(75 + 50)}{(75 + 50 + 200)}$$

$v_C = 1.154 \text{ m/s} \longleftarrow$ ◀

(*b*) *A* dives first and then *B*



x-dir

$$0 = (m_C + m_B) v_{C_2} - m_A v_2 \qquad (2)$$

continued

Relative velocity $v_C - v_{C_2} = 3 \Rightarrow v_2 = v_{C_2} + 3$

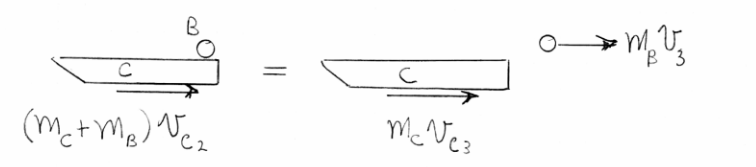
Substitute into (2)

$$0 = \left(m_C + m_B\right) v_{C_2} - m_A\left(v_{C_2} + 3\right)$$

Solve for v_{C_2}

$$v_{C_2} = \frac{-3m_A}{m_A + m_B + m_C} \tag{3}$$

Now look at *C* and *B*.



x-dir. $$\left(m_C + m_B\right) v_{C_2} = m_C\, v_{C_3} + m_B\, v_3 \tag{4}$$

Relative velocity

$$v_3 - v_{C_2} = 3 \Rightarrow v_3 = v_{C_3} + 3$$

Substitute into (4)

$$\left(m_C + m_B\right) v_{C_2} = m_C\, v_{C_3} + m_B\left(v_{C_2} + 3\right)$$

so $$v_{C_3} = \frac{m_C + m_B}{m_C + m_B}\, v_{C_2} - \frac{3m_B}{m_C + m_B} \tag{5}$$

Substituting (3) into (5)

$$v_{C_3} = \frac{-3m_A}{m_A + m_B + m_C} - \frac{3m_B}{m_C + m_B} \tag{6}$$

with numbers

$$v_{C_3} = -3\left[\frac{75}{75 + 50 + 200} + \frac{50}{200 + 50}\right] = -1.2923$$

$v_{C_3} = 1.292$ m/s ← ◀

(*c*) Swimmer *B* dives first – solution is the same as for (*b*) except switch m_A and m_B

$$v_{C_3} = \frac{-3m_B}{m_A + m_B + m_C} - \frac{3m_A}{m_C + m_A}$$

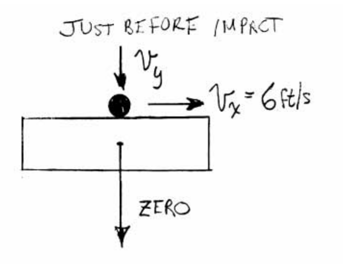
$$= -3\left[\frac{50}{75 + 50 + 200} + \frac{75}{200 + 75}\right] = -1.280$$

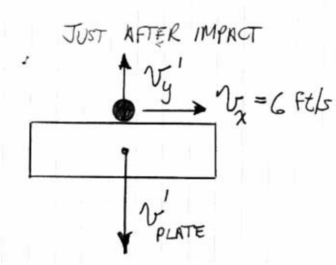
$v_{C_3} = 1.280$ m/s ← ◀

Chapter 13, Solution 150.

$$m_{\text{ball}} = \left(\frac{3}{16}\right)\left(\frac{1}{32.2}\right) = 0.005823$$

$$m_{\text{plate}} = \left(\frac{14}{16}\right)\left(\frac{1}{32.2}\right) = 0.027174$$





$$v_y = \sqrt{2g(4.8)} = 17.582 \text{ ft/s} \qquad v'_y = \sqrt{2g(1.8)} = 10.7666 \text{ ft/s}$$

(*a*) Conservation of momentum $(+y\downarrow)$

$$m_{\text{ball}}\, v_y + 0 = -m_{\text{ball}}\, v'_y + m_{\text{plate}}\, v'_{\text{plate}}$$

$$(0.005823)(17.582) + 0 = -(0.005823)(10.7666) + (0.027174)\, v'_{\text{plate}}$$

$$v'_{\text{plate}} = 6.0747 \text{ ft/s} \qquad v'_{\text{plate}} = 6.07 \text{ ft/s} \downarrow \blacktriangleleft$$

(*b*) Energy loss

Initial energy $$(T+V)_1 = \frac{1}{2}(0.005823)(6)^2 + (0.005823)\,g\,(4.8) = 1.0048$$

Final energy $$(T+V)_2 = \frac{1}{2}(0.005823)(6)^2 + (0.005823)\,g\,(1.8) + \frac{1}{2}(0.027174)(6.0747)^2$$

$$= 0.9437$$

Energy lost $$= (1.0048 - 0.9437) \text{ ft} \cdot \text{lb} = \qquad 0.0611 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

Chapter 13, Solution 151.

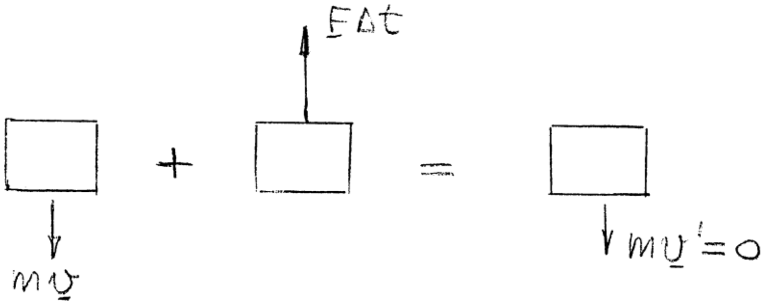
Before impact

$$T_1 = 0,\ V_1 = mgh = (30\text{ kg})(9.81\text{ m/s}^2)(2\text{ m}) = 588.6\text{ N}$$

$$T_2 = \frac{1}{2}mv^2,\ V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2:\quad 588.6 = \frac{1}{2}(30)v^2 \Rightarrow v = 6.2642\text{ m/s}$$

(*a*) Rigid columns



$$+\uparrow\ -mv + F\Delta t = 0$$

$$30(6.2642) = F\Delta t$$

$$F\Delta t = 187.93\text{ N}\cdot\text{s}\ \uparrow\ \text{on the block}$$

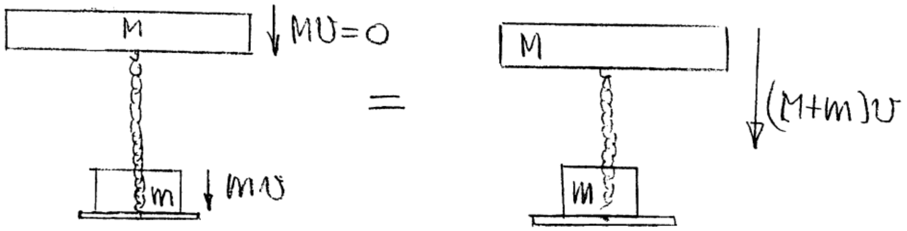
$F\Delta t = 187.9\text{ N}\cdot\text{s}$ ◀

All of the kinetic energy of the block is absorbed by the chain.

$$T = \frac{1}{2}(30)(6.2642)^2 = 588.6\text{ J}$$

$E = 589\text{ J}$ ◀

(*b*) Elastic columns



Momentum of system of block and beam is conserved

$$mv = \left(M + m\right)v' \qquad v' = -\frac{m}{m + M}v = \frac{30}{150}\left(6.2642\right) = 1.2528 \text{ m/s}$$

Referring to figure in Part (*a*) $\quad -mv + F\Delta t = -mv'$

$$F\Delta t = m\left(v - v'\right) = 30\left(6.2642 - 1.2528\right) = 150.34$$

$F\Delta t = 150.3$ N·s ◀

$$E = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 = \frac{30}{2}\left[\left(6.2642\right)^2 - \left(1.2528\right)^2\right] - \frac{120}{2}\left(1.2528\right)^2$$

$$= 565.06 - 94.170 = 470.89$$

$E = 471$ J ◀

Chapter 13, Solution 152.

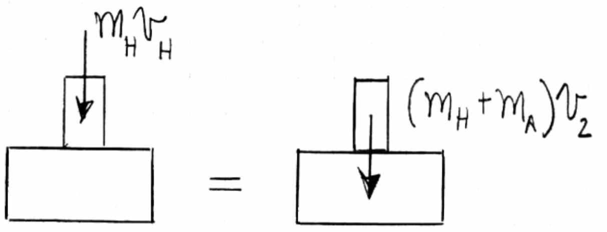
Before impact $T = \frac{1}{2} m_H v_H^2 = \frac{1}{2}(0.75)(6)^2 = 13.5\,\text{J}$

(a) For $m_A = \infty$, $T_2 = 0$ So,

$$\text{Energy absorbed} = 13.5\ \text{J} \blacktriangleleft$$

$$\text{Impulse} = m_H(v_H - \cancelto{0}{v_2}) = (0.75)(6) = 4.5\,\text{N}\cdot\text{s} \blacktriangleleft$$

(b) $m_A = 4\,\text{kg}$



y-dir $\downarrow +$: $m_H v_H = (m_A + m_H) v_2$

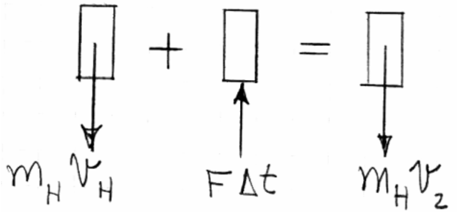
$$v_2 = \frac{m_H v_H}{m_A + m_H} = \frac{(0.75)(6)}{(4 + 0.75)} = 0.9474\ \text{m/s}$$

So $T_2 = \frac{1}{2}(m_A + m_H) v_2^2 = \frac{1}{2}(4 + 0.75)(0.9474)^2 = 2.1316\ \text{J}$

$$\text{Energy absorbed} = T_1 - T_2 = 13.5 - 2.1316$$

$$\Delta E = 11.37\ \text{J} \blacktriangleleft$$

System = hammer



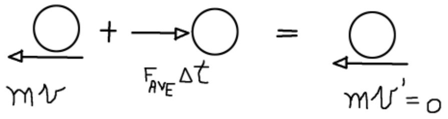
y-dir ↓: $m_H v_H - F\Delta t = m_H v_2$

So $F\Delta t = m_H (v_H - v_2)$

$= 0.75(6 - 0.9474)$

$= 3.79$ $F\Delta t = 3.79$ N·s ◀

Chapter 13, Solution 153.



$$v = 96 \text{ mi/h} = 140.8 \text{ ft/s}$$

$$m = \frac{5}{16}/g = 0.3125/g$$

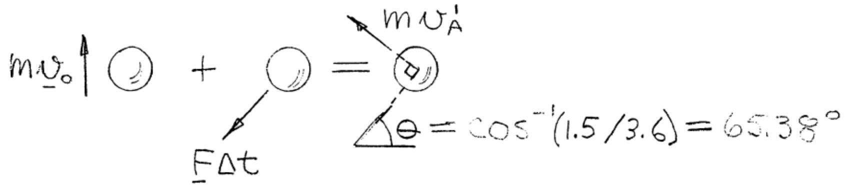
$$t = \frac{d}{v_{\text{AVE}}} = \frac{\frac{8}{12}}{25} = 0.02667 \text{ s}$$

$$\longleftarrow +\frac{0.3125}{g}(140.8) - F_{\text{AVE}}(0.02667) = 0$$

$$F_{\text{AVE}} = 51.2 \text{ lb} \blacktriangleleft$$

Chapter 13, Solution 154.

For the sphere at A' immediately before and after the cord becomes taut



$$mv_0 + F\Delta t = mv_{A'}$$

$$\overset{+}{\nearrow}\theta \quad mv_0 \sin\theta - F\Delta t = 0 \qquad F\Delta t = 0.8 \text{ lb}\cdot\text{s}$$

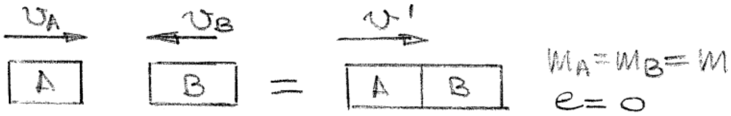
$$m = \frac{4}{g}$$

$$\frac{4}{g}(\sin 65.38°)v_0 = 0.8$$

$$v_0 = 7.08 \text{ ft/s} \blacktriangleleft$$

Chapter 13, Solution 155.

(*a*)



Conservation of total momentum

$$\xrightarrow{+} \quad mv_A - mv_B = 2mv'$$

$$v' = \frac{1}{2}(v_A - v_B) \blacktriangleleft$$

(*b*) Energy loss

$$E_L = T_A + T_B - (T'_A + T'_B)$$

$$E_L = \frac{1}{2}m\left(v_A^2 + v_B^2\right) - \frac{1}{2}m\left(v'^2 + v'^2\right)$$

From (*a*)

$$v' = \frac{1}{2}(v_A - v_B)$$

$$E_L = \frac{1}{2}m\left(v_A^2 + v_B^2\right) - \frac{1}{2}m\left[\frac{1}{2}(v_A - v_B)^2\right]$$

$$E_L = \frac{1}{2}m\left(v_A^2 + v_B^2\right) - \frac{1}{4}m\left(v_A^2 - 2v_Av_B + v_B^2\right)$$

$$E_L = \frac{1}{4}m\left[v_A^2 + 2v_Av_B + v_B^2\right] = \frac{1}{4}m(v_A + v_B)^2 \blacktriangleleft$$

Chapter 13, Solution 156.

Before impact After impact

$$\cancel{m}\, v_A + \cancel{m}\, v_B = \cancel{m}\, v'_A + \cancel{m}\, v'_B \qquad (1)$$

$$v'_B - v'_A = e\,(v_A - v_B) \qquad (2)$$

From (1) and (2) solve for v'_A, v'_B $\quad v'_A = \dfrac{(v_A - v_B) - 0.5\,(v_A - v_B)}{2}$

$$v'_B = \frac{(v_A + v_B) + 0.5(v_A - v_B)}{2}$$

(a) $v'_A = (v_A + 3v_B)/4$ ◀

$v'_B = (3v_A + v_B)/4$ ◀

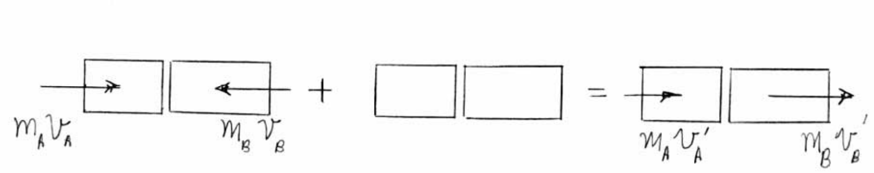
$$(b)\text{ Loss of energy } = \frac{m}{2}\,(v_A^2 + v_B^2) - \frac{m}{2}\,(v_A'^2 + v_B'^2)$$

$$\text{Loss of energy} = \frac{m}{2}\left[v_A^2 + v_B^2 - \frac{1}{16}(v_A^2 + 6v_A v_B + 9v_B^2 + 9v_A^2 + 6v_A v_B + v_B^2)\right]$$

Loss of energy $= \dfrac{3m}{16}(v_A - v_B)^2$ ◀

Chapter 13, Solution 157.

System = $A + B$



(a) x-dir

$$m_A v_A - m_B v_B = m_A v_A' + m_B v_B' \tag{1}$$

Unknowns v_B', v_A'

Coefficient of restitution $\xrightarrow{}+$

$$v_{Br}' - v_{Ar}' = e(v_{Ar} - v_{Br})$$

For our problem

$$v_B' - v_A' = e(v_A + v_B) \tag{2}$$

With numbers $m_A = 0.6;\quad m_B = 0.9;\quad v_A = 4\text{ m/s};\quad v_B = 2\text{ m/s}$

Solve 2 equations and 2 unknowns

$$v_A' = -2.3\text{ m/s};\qquad v_B' = 2.2\text{ m/s}$$

$v_A' = 2.3\text{ m/s} \longleftarrow$ ◀

$v_B' = 2.2\text{ m/s} \longrightarrow$ ◀

(b) Energy lost

$$T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.6)(4)^2 + \frac{1}{2}(0.9)(2)^2 = 6.6\text{ J}$$

$$T_2 = \frac{1}{2}m_A\left(v_A'\right)^2 + \frac{1}{2}m_B\left(v_B'\right)^2 = \frac{1}{2}(0.6)(2.3)^2 + \frac{1}{2}(0.9)(2.2)^2 = 3.765\text{ J}$$

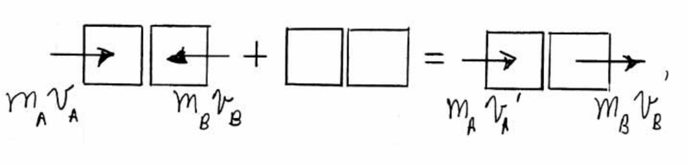
$$\Delta E = T_1 - T_2 = 2.835\text{J}$$

$\Delta E = 2.84\text{ J}$ ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 158.

System = $A + B$



x-dir

$$m_A v_A - m_B v_B = m_A v_A' + m_B v_B' \quad (1)$$

Unknowns e, v_A'

Coefficient of restitution

$$v_B' - v_A' = e(v_A + v_B) \quad (2)$$

Where, $v_A = 4$ m/s; $v_B = 2$ m/s; $v_B' = 2.5$ m/s

$m_A = 0.6$ kg; $m_B = 0.9$ kg

Solve 2 equations and 2 unknowns

$$v_A' = -2.75 \text{ m/s}; \; e = 0.875$$

$e = 0.875$ ◀

Chapter 13, Solution 159.

From conservation of momentum

$$\xrightarrow{+} m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$\frac{1.2}{gv_A} - \frac{2.4}{g} = -\frac{1.2}{gv'_A} + 0 \qquad g\text{'s cancel} \tag{1}$$

From restitution

$$0.8 = \frac{v'_A}{v_A + 18}, \quad v'_A = 0.8 v_A + 14.4 \tag{2}$$

(*a*) Velocity of *A* before impact from equations (1) and (2)

$$1.2 v_A - 43.2 = -1.2(0.8 v_A + 14.4) = -0.96 v_A - 17.28$$

$$2.16 v_A = 25.92$$

$$v_A = 12 \text{ ft/s} \longrightarrow$$ ◀

(*b*) Velocity of *A* after impact

$$v'_A = 0.8(12) + 14.4$$

$$v'_A = 24 \text{ ft/s} \longleftarrow$$ ◀

Chapter 13, Solution 160.

From conservation of momentum

$$\xrightarrow{+}\ m_A v_A + m_B v_B = m_A v_B' + m_B v_B' \quad g\text{'s cancel}$$

$$\left(\frac{1.2}{g}\right)(24\text{ ft/s}) - \left(\frac{2.4}{g}\right)v_B = 0 + \left(\frac{2.4}{g}\right)v_B' \qquad (1)$$

From restitution

$$0.2 = \frac{v_B'}{24 + v_B}, \quad v_B' = 4.8 + 0.2v_B \qquad (2)$$

(*a*) Velocity of *B* before impact from equations (1) and (2)

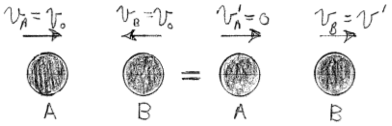
$$28.8 - 2.4v_B = 2.4(4.8 + 0.2v_B) = 11.52 + 0.48v_B$$

$$2.88v_B = 17.28 \qquad v_B = 6\text{ ft/s} \leftarrow \blacktriangleleft$$

(*b*) Velocity of *B* after impact

$$v_B' = 4.8 + 0.2(6) \qquad v_B' = 6\text{ ft/s} \rightarrow \blacktriangleleft$$

Chapter 13, Solution 161.



(*a*) Total momentum conserved

$$\xrightarrow{+} \quad m_A v_A + m_B v_B = m_A v_A' + m_B v'$$

$$(6\text{ kg})v_0 + m_B(-v_0) = 0 + m_B v' \Rightarrow v' = \left(\frac{6}{m_B - 1}\right)v_0 \qquad (1)$$

Relative velocities

$$\xrightarrow{+} \quad (v_A - v_B)e = v_B' - v_A' \Rightarrow v' = 2v_0 e \qquad (2)$$

From equations (1) and (2)

$$2v_0 e = \left(\frac{6}{m_B - 1}\right)v_0 \Rightarrow 2v_0(0.5) = \left(\frac{6}{m_B - 1}\right)v_0$$

$$m_B = 3\text{ kg} \blacktriangleleft$$

(*b*) Using $\quad 2v_0 e = \left(\frac{6}{m_B - 1}\right)v_0$

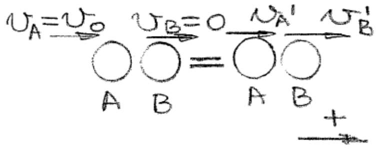
Gives, $\quad 2e + 1 = \frac{6}{m_B} \Rightarrow m_B = \frac{6}{2e + 1}$

$$e = 0,\ m_B = 6\text{ kg}$$

$$e = 1,\ m_B = 2\text{ kg}$$

$$2\text{ kg} \le m_B \le 6\text{ kg} \blacktriangleleft$$

Chapter 13, Solution 162.



(*a*) First collision (between *A* and *B*)

The total momentum is conserved

$$mv_A + mv_B = mv_A' + mv_B'$$

$$v_0 = v_A' + v_B' \tag{1}$$

Relative velocities

$$(v_A - v_B)e = (v_B' - v_A')$$

$$v_0 e = v_B' - v_A' \tag{2}$$

Solving equations (1) and (2) simultaneously

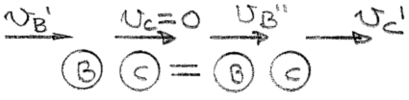
$$v_A' = \frac{v_0(1-e)}{2} \blacktriangleleft$$

$$v_B' = \frac{v_0(1+e)}{2} \blacktriangleleft$$

(*b*) Second collision (Between *B* and *C*)

The total momentum is conserved.

$$mv_B' + mv_C = mv_B'' + mv_C'$$



Using the result from (*a*) for v_B'

$$\frac{v_0(1+e)}{2} + 0 = v_B'' + v_C' \tag{3}$$

Relative velocities

$$(v_B' - 0)e = v_C' - v_B''$$

Substituting again for v_B' from (*a*)

$$v_0 \frac{(1+e)}{2}(e) = v_C' - v_B'' \tag{4}$$

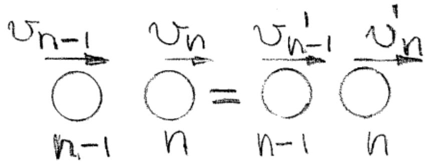
continued

Solving equations (3) and (4) simultaneously

$$v_C' = \frac{1}{2}\left[\frac{v_0(1+e)}{2} + v_0(1+e)\frac{(e)}{2}\right]$$

$$v_C' = \frac{v_0(1+e)^2}{4} \blacktriangleleft$$

$$v_B'' = \frac{v_0(1-e)^2}{4} \blacktriangleleft$$



(*c*) For *n* spheres

n Balls

$n-1th$ collision

We note from the answer to part (*b*), with $n = 3$

$$v_n' = v_3' = v_C' = \frac{v_0(1+e)^2}{4}$$

or $$v_3' = \frac{v_0(1+e)^{(3-1)}}{2^{(3-1)}}$$

Thus for *n* balls

$$v_n' = \frac{v_0(1+e)^{(n-1)}}{2^{(n-1)}} \blacktriangleleft$$

(*d*) For $n = 8,\ e = 0.90$

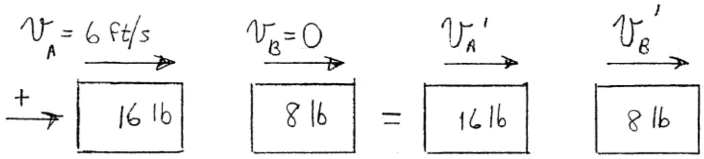
From the answer to part (*c*) with $n = 8$

$$v_B' = \frac{v_0(1+0.9)^{(8-1)}}{2^{(8-1)}} = \frac{v_0(1.9)^7}{(2)^7}$$

$$v_8' = 0.698v_0 \blacktriangleleft$$

Chapter 13, Solution 163.

(*a*) Packages *A* and *B*



Total momentum conserved

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$\frac{16}{g}(6) = \frac{16}{g}v_A' + \frac{8}{g}v_B' \Rightarrow 16v_A' + 8v_B' = 96$$

$$2v_A' + v_B' = 12 \;\; (1)$$

Relative velocities

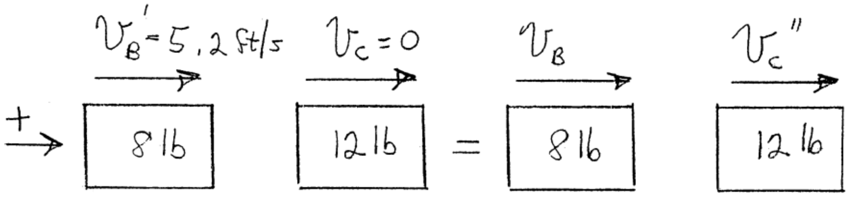
$$(v_A - v_B)e = v_B' - v_A' \Rightarrow v_B' - v_A' = 0.3(6) = 1.8 \qquad (2)$$

Solving Equations (1) and (2) simultaneously

$$v_A' = 3.4 \text{ ft/s} \longrightarrow$$

$$v_B' = 5.2 \text{ ft/s} \longrightarrow$$

Packages *B* and *C*



$$m_B v_B' + m_C v_C = m_B v_B'' + m_C v_C''$$

$$\frac{8}{g}(5.2) = \frac{8}{g}v_B'' + \frac{12}{g}v_C' \qquad 4v_B'' + 6v_C' = 20.8 \qquad (3)$$

Relative velocities

$$(v_B' - v_C)e = v_C' - v_B'' \Rightarrow v_C' - v_B'' = 0.3(5.2) = 1.56 \qquad (4)$$

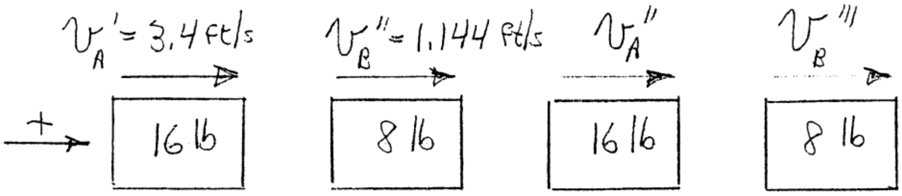
continued

Solving (3) and (4) simultaneously

$$v'_C = 2.70 \text{ ft/s} \longrightarrow ◀$$

$$v''_B = 1.144 \text{ ft/s} \longrightarrow$$

(*b*) Packages *A* and *B* (second time)



$$m_A v'_A + m_B v''_B = m_A v''_A + m_B v'''_B$$

$$\frac{16}{g}(3.4) + \frac{8}{g}(1.144) = \frac{16}{g}v''_A + \frac{8}{g}v''_B;\ 2v''_A + v'''_B = 7.944 \qquad (5)$$

$$(v'_A - v''_B)e = v'''_B - v'_A$$

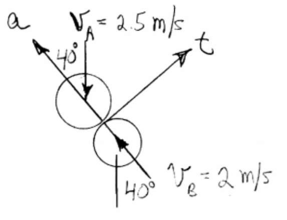
$$(3.4 - 1.144)(0.3) = 0.6768 = v'''_B - v_A;\ -v_A + v_B = 0.6768 \qquad (6)$$

Solving Equations (5) and (6) simultaneously

$$v''_A = 2.42 \text{ ft/s} \longrightarrow ◀$$

Chapter 13, Solution 164.

Impact


n
$v_A = 2.5$ m/s
t
40°
40°
$v_B = 2$ m/s

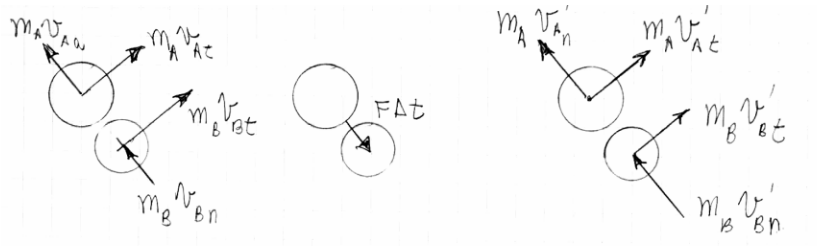
$$v_{An} = -2.5\cos 40° = -1.915 \text{ m/s}$$

$$v_{At} = -2.5\sin 40° = -1.607 \text{ m/s}$$

$$v_{Bn} = 2 \text{ m/s}$$

$$v_{Bt} = 0$$

Impulse-momentum


$m_A v_{An}$
$m_A v_{At}$
$m_B v_{Bt}$
$m_B v_{Bn}$
$F\Delta t$
$m_A v'_{An}$
$m_A v'_{At}$
$m_B v'_{Bt}$
$m_B v'_{Bn}$

Unknowns $v'_{An}, v'_{Bn}, v'_{At}, v'_{Bt}$

System $A + B$ n-dir ↖

$$m_A v_{An} + m_B v_{Bn} = m_A v'_{An} + m_B v'_{Bn} \tag{1}$$

Coefficient of restitution

$$v'_{Bn} - v'_{An} = e\left(v_{An} - v_{Bn}\right) \tag{2}$$

Solve (1) and (2) for $v'_{An} + v'_{Bn}$

$$v'_{An} = 0.7493 \text{ m/s}; \quad v'_{Bn} = -2.1870 \text{ m/s}$$

System A t-dir ↗

$$m_A v_{At} = m_A v'_{At} \Rightarrow v'_{At} = -1.607 \text{ m/s}$$

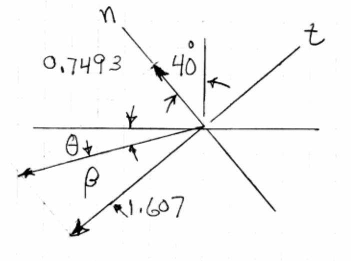
continued

System B t-dir ↗

$$m_B v_{Bt} = m_B v'_{Bt} \Rightarrow v'_{Bt} = 0 \text{ m/s}$$

Resolve into components

Ball A



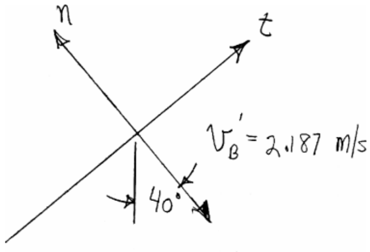
$$v'_A = \sqrt{(0.7493)^2 + (1.607)^2} = 1.773 \text{ m/s}$$

$$\beta = \tan^{-1}\left(\frac{0.7493}{1.607}\right) = 25.0°$$

$$\theta = 40° - 25° = 15.0°$$

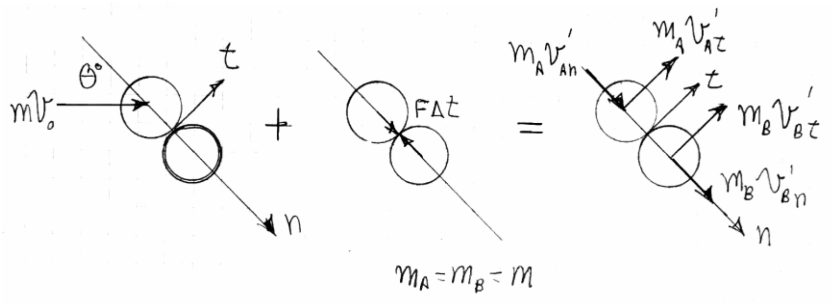
So $\vec{v}'_A = 1.773$ m/s ↙ 15° ◀

Ball B



$\vec{v}'_B = 2.19$ m/s ↘ 50° ◀

Chapter 13, Solution 165.



Ball *A t-dir*

$$\cancel{m}v_0\sin\theta = \cancel{m}v'_{At} \Rightarrow v'_{At} = v_0\sin\theta$$

Ball *B t-dir*

$$0 = m_B v'_{Bt} \Rightarrow v'_{Bt} = 0$$

Ball *A* + *B n-dir*

$$\cancel{m}v_0\cos\theta + 0 = \cancel{m}v'_{An} + \cancel{m}v'_{Bn} \tag{1}$$

Coefficient of restitution

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

$$v'_{Bn} - v'_{An} = e(v_0\cos\theta - 0) \tag{2}$$

Solve (1) and (2)

$$v'_{An} = v_0\left(\frac{1-e}{2}\cos\theta\right);\quad v'_{Bn} = v_0\left(\frac{1+e}{2}\right)\cos\theta$$

With numbers

$$e = 0.8;\quad \theta = 45°$$

$$v'_{At} = v_0\sin 45° = 0.707\,v_0$$

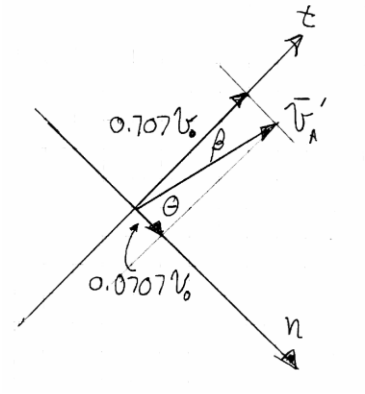
$$v'_{An} = v_0\left(\frac{1-0.8}{2}\cos 45°\right) = 0.0707\ v_0$$

$$v'_{Bt} = 0$$

$$v'_{Bn} = v_0\left(\frac{1+0.8}{2}\right)\cos 45° = 0.6364\,v_0$$

continued

(*A*)


t
0.707v₀
v'_A
β
θ
0.0707v₀
n

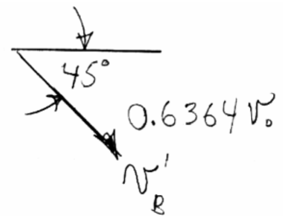
$$|v''_A| = \left[(0.707\,v_0)^2 + (0.0707 v_0)^2\right]^{\frac{1}{2}}$$

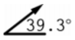
$$= 0.711 v_0$$

$$\beta = \tan^{-1}\left(\frac{0.0707}{0.707}\right) = 5.7106°$$

So $\theta = 45 - 5.7106 = 39.3°$

(*B*)

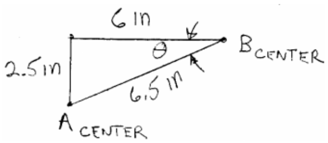

45°
0.6364v₀
v'_B

$\vec{v}'_A = 0.711 v_0$  ◀

$\vec{v}'_B = 0.636 v_0$ ◀

Chapter 13, Solution 166.

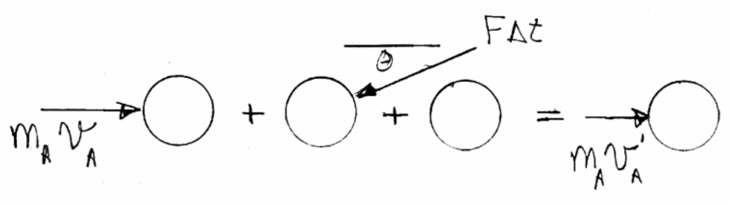
Angle of impulse force from geometry


6 in
θ
2.5 in
6.5 in
B CENTER
A CENTER

$$\theta = \cos^{-1}\frac{6}{6.5} = 22.62°$$

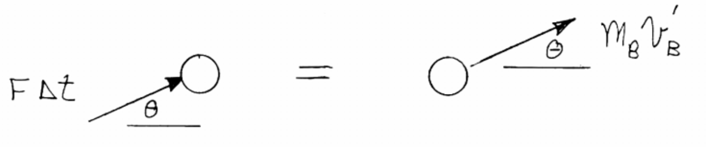
Total momentum conserved

Ball A:


FΔt
θ
$m_A v_A$
+
+
=
$m_A v_A'$

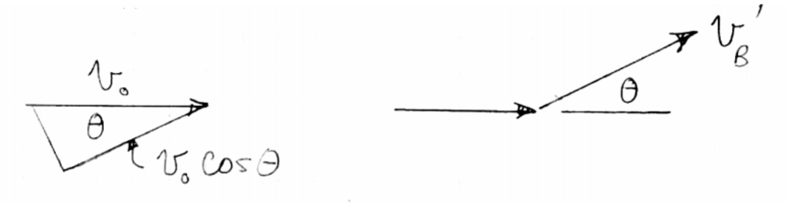
$$x \xrightarrow{+}: m_A v_A - F\Delta t(\cos\theta) + 0 = m_A v_A' \qquad (1)$$

Ball B:



$$F\Delta t = m_B v_B'$$

Restitution $(v_A = v_0)$


v_0
θ
$v_0 \cos\theta$
v_B'
θ

continued

Approach Separation

$$v'_B - v'_A\cos\theta = e\left(v_0\cos\theta\right); \quad v'_B = \left(v'_A + ev_0\right)\cos\theta$$

Using equations (1) and (2) $x \xrightarrow{+}: m_A v_A = m_B v'_B\cos\theta + m_A v'_A$

$$\left(17.5/g\right)\left(6\text{ ft/s}\right) = \left(1.6/g\right)v'_B\left(\frac{6}{6.5}\right) + \left(17.5/g\right)\mathrm{v}'_A$$

g's cancel

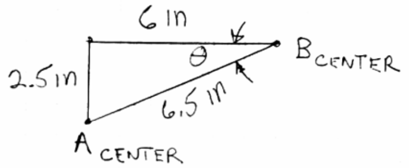
Substituting for v'_B; $105 = 1.6\left(\frac{6}{6.5}\right)\left[4.8\left(\frac{6}{6.5}\right) + v'_A\left(\frac{6}{6.5}\right)\right] + 17.5v'_A$

$v'_A = 5.22$ ft/s → ◀

$v'_B = 9.25$ ft/s ∠ 22.6°◀

Chapter 13, Solution 167.

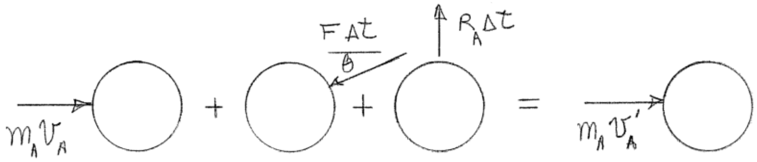
Angle of impulse force from geometry


6 in
θ
B CENTER
2.5 in
6.5 in
A CENTER

$$\theta = \cos^{-1}\frac{6}{6.5} = 22.62^\circ$$

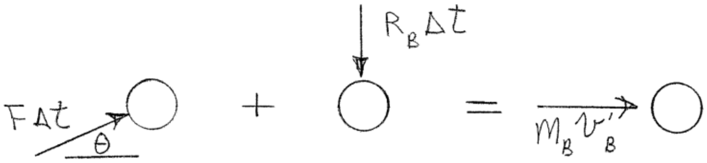
Total momentum conserved

Ball A:


$m_A v_A$
FΔt
θ
$R_A \Delta t$
$m_A v_A'$

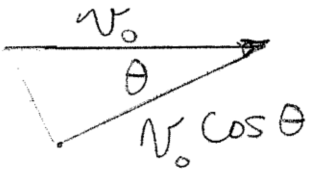
$$x \xrightarrow{+}: \; m_A v_A - F\Delta t(\cos\theta) + 0 = m_A v_A' \tag{1}$$

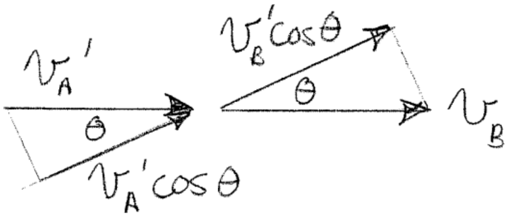
Ball B:


FΔt
θ
$R_B \Delta t$
$m_B v_B'$

$$x \xrightarrow{+}: \; F\Delta t\cos\theta = m_B v_B' \tag{2}$$

Restitution $(v_A - v_0)$


v_0
θ
$v_0 \cos\theta$


v_A'
θ
$v_A' \cos\theta$
$v_B' \cos\theta$
θ
v_B'

continued

Approach Separation

$$v_B'\cos\theta - v_A'\cos\theta = e\left(v_A\cos\theta\right);\qquad v_B' - v_A' = 4.8 \qquad (3)$$

Using Equations (1) and (2) $x \xrightarrow{+}:\ m_A v_A = m_B v_B' + m_A v_A'$

$$\left(17.5/g\right)\left(6\right) = \left(1.6/g\right)v_B' + \left(17.5/g\right)v_A'$$ g's cancel

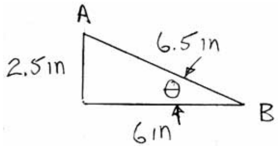
Substituting for v_B' from (3) $105 = 1.6\left(4.8 + v_A'\right) + 17.5v_A'$

$$v_A' = 5.10 \text{ ft/s} \longrightarrow \blacktriangleleft$$

$$v_B' = 9.90 \text{ ft/s} \longrightarrow \blacktriangleleft$$

Chapter 13, Solution 168.

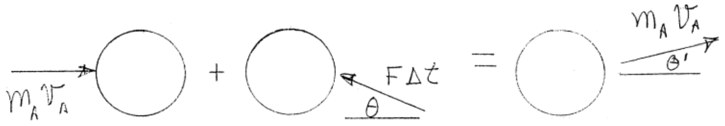
Angle of impulse force from geometry of *A* and *B*


A
2.5 in
6.5 in
θ
B
6 in

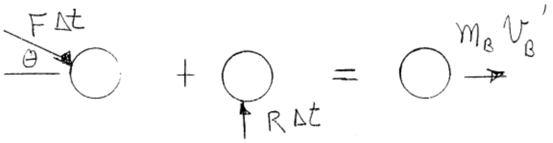
$$\theta = \cos^{-1}\left(\frac{6}{6.5}\right) = 22.62°$$

Total momentum conserved

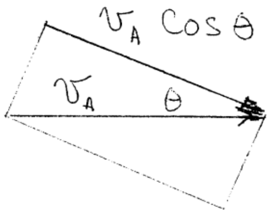
Ball A:


$m_A v_A$
+
$F\Delta t$
θ
=
$m_A v_A'$
θ'

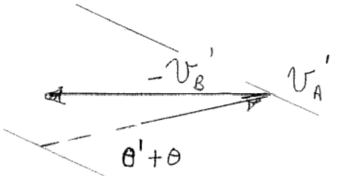
Ball B:


$F\Delta t$
θ
+
$R\Delta t$
=
$m_B v_B'$

Restitution


$v_A \cos\theta$
v_A
θ

Approach



Separation

$$e = \frac{v_{Bn}' - v_{An}'}{v_{An} - v_{Bn}} \Rightarrow e = 1 = \frac{-v_A'\cos\left(\theta' + 22.6°\right) + v_B'\left(\frac{6}{6.5}\right)}{6\left(\frac{6}{6.5}\right)} \tag{1}$$

$$A: \quad \cancel{m_A} v_A \sin\theta = \cancel{m_A} v_A' \sin\left(\theta' + 22.6°\right)$$

$$6\left(\frac{2.5}{6.5}\right) = v_A' \sin\left(\theta' + 22.6°\right) \tag{2}$$

continued

$A + B$:
$$m_A v_A = m_A v_A' \cos\theta' + m_B v_B'$$

$$(58/g)(6) = (58/g)v_A' \cos\theta' + (5.3/g)v_B' \quad g\text{'s cancel}$$

Equations (1), (2), and (3) in v_A', v_B' and θ'

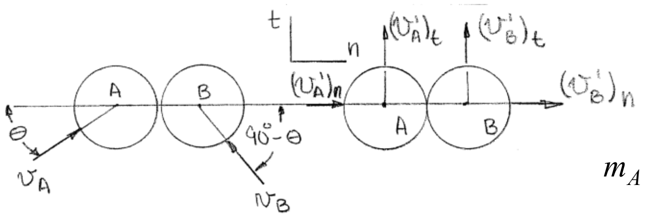
$$v_A' = 5.027 \text{ ft/s}; \quad v_B' = 10.838 \text{ ft/s}; \quad \theta' = 0.08218 \text{ rad} = 4.71°$$

$$v_A' = 5.03 \text{ ft/s} \measuredangle 4.71° \blacktriangleleft$$

$$v_B' = 10.84 \text{ ft/s} \longrightarrow \blacktriangleleft$$

Chapter 13, Solution 169.

(*a*)



$m_A = m_B = m$

Before After

t-Direction

Momentum of *A* is conserved.

$$mv_A \sin\theta = m(v_A')_t$$

$$(v_A')_t = v_A \sin\theta$$

Momentum of *B* is conserved.

$$mv_B \cos\theta = m(v_B')_t$$

$$(v_B')_t = v_B \cos\theta$$

n-Direction

Total momentum is conserved.

$$mv_A \cos\theta - mv_B \sin\theta = m(v_A')_n + m(v_B')_n$$

$$(v_A')_n + (v_B')_n = v_A \cos\theta - v_B \sin\theta \quad (1)$$

Relative velocities (coefficient of restitution)

$$e = 1 \qquad (v_B')_n - (v_A')_n = (1)(v_A \cos\theta + v_B \sin\theta) \quad (2)$$

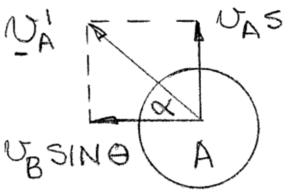
Adding Equation (1) and (2)

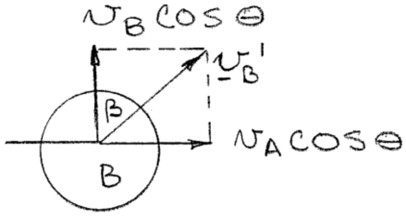
$$(v_B')_n = v_A \cos\theta$$

(1) − (2) $$(v_A')_n = -v_B \sin\theta$$

continued

Thus, after impact


v_A'
$v_A \sin\theta$
α
$v_B \sin\theta$
A


$v_B \cos\theta$
v_B'
β
$v_A \cos\theta$
B

$$\tan\alpha = \frac{v_A}{v_B} \qquad \tan\beta\frac{v_A}{v_B}$$

Thus $\alpha = \beta$ and $v_A' \perp v_B'$ ◀

(*b*) Using the results from (*a*)

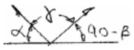
$$v_A' = \sqrt{(v_A')_t^2 + (v_A')_n^2} = \sqrt{v_A^2 \sin^2\theta + v_B^2 \sin^2\theta}$$

$$v_A' = \sin 30^\circ\sqrt{(30)^2 + (40)^2} = 25 \text{ ft/s}$$ ◀

$$v_B' = \sqrt{(v_B')_t^2 + (v_B')_n^2} = \sqrt{v_B^2 \cos^2\theta + v_A^2 \cos^2\theta}$$

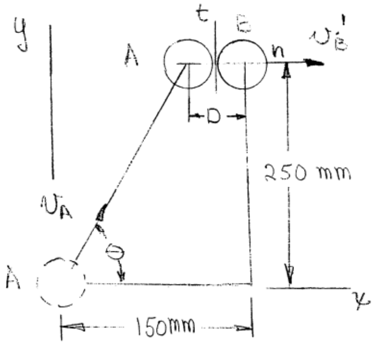
$$v_B' = \cos 30^\circ\sqrt{(40)^2 + (30)^2} = 43.3 \text{ ft/s}$$ ◀

$$\alpha = \beta = \tan^{-1}\frac{v_A}{v_B} = \tan^{-1}\frac{30}{40} = 36.9^\circ$$


α
γ
90-β

$$\gamma = 180 - (\alpha + 90 - \beta)\big] = 90^\circ$$ ◀

Chapter 13, Solution 170.



(*a*) Since v_B' is in the *x*-direction and (assuming no friction), the common tangent between *A* and *B* at impact must be parallel to the *y*-axis

Thus $$\tan\theta = \frac{250}{150 - D}$$

$$\theta = \tan^{-1}\frac{250}{150 - 60} = 70.20^\circ$$

$$\theta = 70.2^\circ \blacktriangleleft$$

(*b*) Conservation of momentum in *x*(*n*) direction

$$mv_A\cos\theta + m(v_B)_n = m(v_A')_n + mv_B'$$

$$(1)\cos(70.20) + 0 = (v_A')_n + v_B'$$

$$0.3387 = (v_A')_n + (v_B') \qquad (1)$$

Relative velocities in the *n* direction

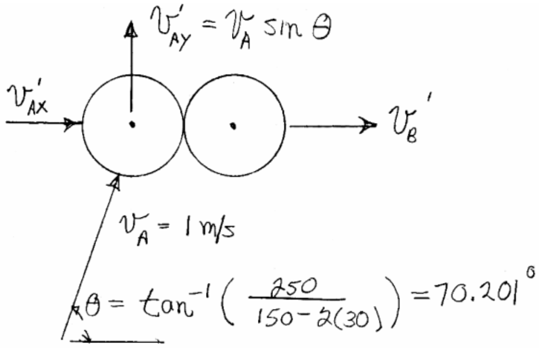
$$e = 0.9 \qquad (v_A\cos\theta - (v_B)_n)e = v_B' - (v_A')_n$$

$$(0.3387 - 0)(0.9) = v_B' - (v_A')_n \quad (2)$$

(1) + (2)

$$2v_B' = 0.3387(1.9) \qquad v_B' = 0.322 \text{ m/s} \blacktriangleleft$$

Chapter 13, Solution 171.


$v'_{Ay} = v_A \sin\theta$
v'_{Ax}
v_B'
$v_A = 1$ m/s
$\theta = \tan^{-1}\left(\frac{250}{150-2(30)}\right) = 70.201°$

Momentum: $v_A \cos\theta + 0 = v_{Ax} + v'_B$

Restitution: $v'_B - v'_{Ax} = 0.9\, v_A \cos\theta$

$$\left(v_A \cos\theta - v'_{Ax}\right) - v'_{Ax} = 0.9\, v_A \cos\theta$$

$$v'_{Ax} = \frac{1}{2}\left[0.1(1)\cos 70.2°\right] = 0.016936 \text{ m/s}$$

$$v'_{Ay} = (1)\sin 70.2° = 0.94089, \;\; v'_B = 0.32178$$

(*a*) $v'_A = 0.941$ m/s 89° ◀

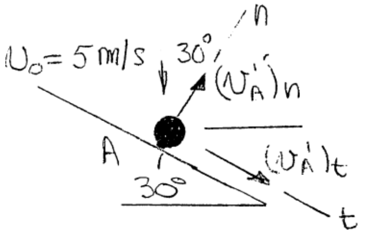
(*b*) Fraction of Initial Energy loss = F. L.

$$\text{F. L.} = \frac{\cancel{\frac{1}{2}}\cancel{m}(1)^2 - \cancel{\frac{1}{2}}\cancel{m}\left(v'_B\right)^2 - \cancel{\frac{1}{2}}\cancel{m}\left(v'_A\right)^2}{\cancel{\frac{1}{2}}\cancel{m}(1)^2}$$

$$\text{F. L.} = 1 - 0.1035 - 0.8856 = 0.01090$$ ◀

Chapter 13, Solution 172.

Rebound at A



Conservation of momentum – t-direction

$$mv_0 \sin 30° = m(v'_A)_t \Rightarrow (v'_A)_t = 5\sin 30°$$

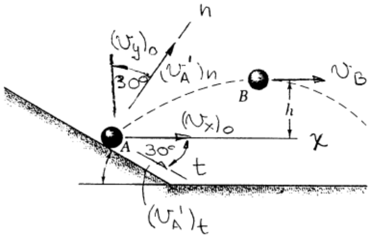
$$(v'_A)_t = 2.5 \text{ m/s}$$

Relative velocities in the n-direction

$$(-v_0 \cos 30° - 0)e = 0 - (v'_A)_n \Rightarrow (v'_A)_n = (5\cos 30)(0.8)$$

$$(v'_A)_n = 3.464 \text{ m/s}$$

Projectile motion between A and B



After rebound

$$(v_x)_0 = (v'_A)_t \cos 30° + (v'_A)_n \sin 30°$$

$$= 2.5\cos 30° + 3.464\sin 30°$$

$$(v_x)_0 = 3.897 \text{ m/s}$$

$$(v_y)_0 = -(v'_A)_t \sin 30° + (v'_A)_n \cos 30°$$

$$= -2.5\sin 30° + 3.464\cos 30°$$

$$(v_y)_0 = 1.75 \text{ m/s}$$

x-direction: $x = (v_x)_0 t = 3.897t,\ v_x = (v_x)_0 = 3.897$ m/s

y-direction: $y = (v_y)_0 t - \frac{1}{2}gt^2 = 1.75t - 4.905t^2$

$$v_y = (v_y)_0 - gt = 1.75 - 9.81t$$

At B: $v_y = 0 = 1.75 - 9.81t_{A-B} \Rightarrow t_{A-B} = 0.17839$ s

$$y = h = (v_y)_0 t_{A-B} - 4.905t_{A-B}^2 = 1.75(0.17839) - 4.905(0.17839)^2$$

$$h = 0.15609 \text{ m}$$

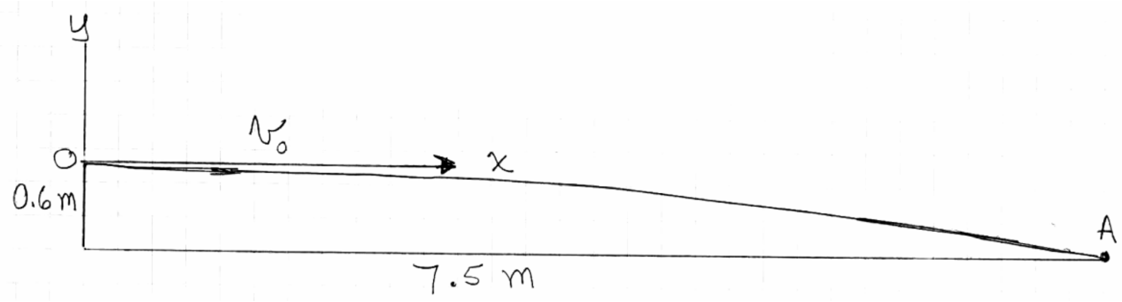
$$h = 156.1 \text{ mm} \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

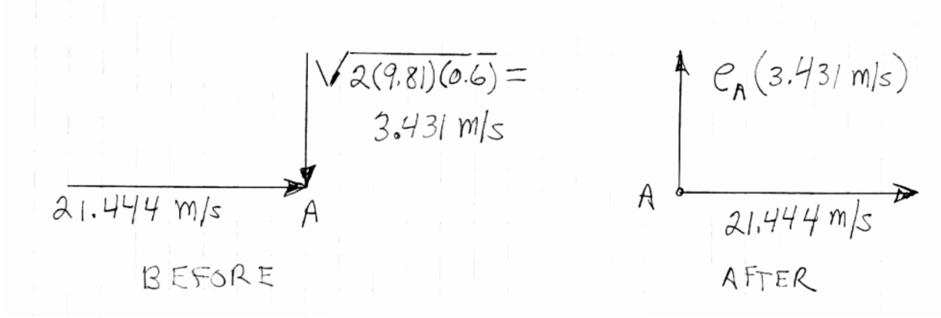
Chapter 13, Solution 173.

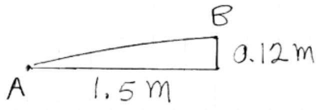
$$x_A = v_0 t_A = 7.5 \text{ m}, \quad \frac{g t_A^2}{2} = 0.6 \text{ m}, \quad \frac{9.81}{2} \text{ m/s}^2 \, t_A^2 = 0.6 \text{ m} \Rightarrow t_A = 0.34975 \text{ s}$$

$$v_0 = 21.444 \text{ m/s}$$


y
O
v_0
x
0.6 m
A
7.5 m

(*a*) First bounce:


$\sqrt{2(9.81)(0.6)} =$
3.431 m/s
21.444 m/s
A
BEFORE
e_A(3.431 m/s)
A
21.444 m/s
AFTER



$$v_0 t_B = 1.5 \text{ m}, \quad t_B = 0.06995 \text{ s}$$

$$0.12 \text{ m} = e_A (3.431 \text{ m}) t_B - \frac{9.81 \text{ m/s}^2}{2} t_B^2$$

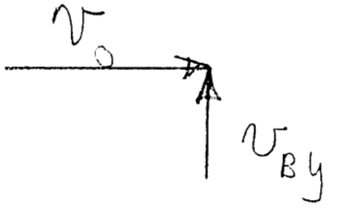
$$0.12 = e_A (3.431)(0.06995) - (4.905)(0.06995)^2$$

$$= 0.24 e_A - 0.024$$

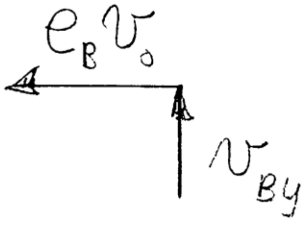
$$e_A = 0.600 \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

(*b*) Second bounce:



Before

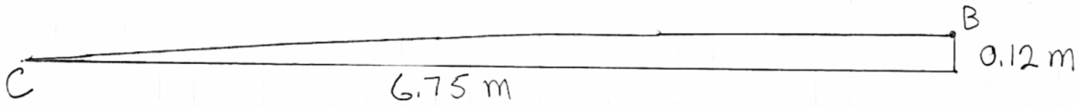


After

$$v_{By} = 0.6(3.431) - 9.81t_B$$

$$= 1.3724 \text{ m/s}$$

$$e_B v_0 = e_B(21.444 \text{ m/s})$$



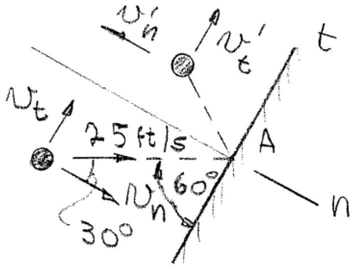
$$y_C = 0.12 + v_{By}t_C - 4.905t_C^2 = 0 = 0.12 + 1.3724t_C - 4.905t_C^2$$

$$t_C = 0.3497 \text{ s}$$

$$x_C = 6.75 = e_B v_0 t_C = e_B(21.444)(0.3497)$$

$$e_B = 0.900 \blacktriangleleft$$

Chapter 13, Solution 174.


v_n'
v_t'
t
v_t
25 ft/s
A
60°
v_n
30°
n

Momentum in *t* direction is conserved

$$mv\sin 30° = mv_t'$$

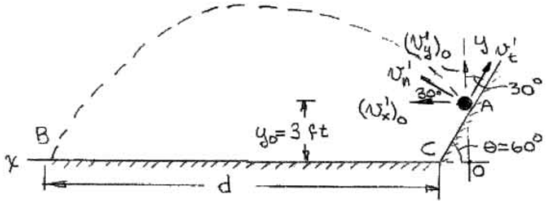
$$(25)(\sin 30°) = v_t'$$

$$v_t' = 12.5 \text{ ft/s}$$

Coefficient of restitution in *n*-direction

$$(v\cos 30°)e = v_n'$$

$$(25)(\cos 30°)(0.9) = v_n' \qquad v_n' = 19.49 \text{ ft/s}$$


$(v_y')_0$
y
v_t'
v_n'
30°
30°
A
$(v_x')_0$
$y_0 = 3$ ft
B
x
C
$\theta = 60°$
O
d

Write v' in terms of *x* and *y* components

$$(v_x')_0 = v_n'(\cos 30°) - v_t'(\sin 30°) = 19.49(\cos 30°) - 12.5(\sin 30°)$$

$$= 10.63 \text{ ft/s}$$

$$(v_y')_0 = v_n'(\sin 30°) + v_t'(\cos 30°) = 19.49(\sin 30°) + 12.5(\cos 30°)$$

$$= 20.57 \text{ ft/s}$$

Projectile motion

$$y = y_0 + \left(v'_y\right)_0 t - \frac{1}{2}gt^2 = 3\text{ ft} + \left(20.57\text{ ft/s}\right)t - \left(32.2\text{ ft/s}^2\right)\frac{t^2}{2}$$

At *B*,

$$y = 0 = 3 + 20.57t_B - 16.1t_B^2;\ t_B = 1.4098\text{ s}$$

$$x_B = x_0 + \left(v'_x\right)_0 t_B = 0 + 10.63\left(1.4098\right);\ x_B = 14.986\text{ ft}$$

$$d = x_B - 3\cos 60° = \left(14.986\text{ ft}\right) - \left(3\text{ ft}\right)\cot 60° = 13.254\text{ ft}$$

$d = 13.25$ ft ◀

Chapter 13, Solution 175.

Find x and y components of v

$$\overline{v} = v_x\mathbf{i} - v_y\mathbf{j}$$

After the first impact x component is multiplied by e and the y component is unchanged

$$\overline{v}' = -ev_x\mathbf{i} - v_y\mathbf{j}$$

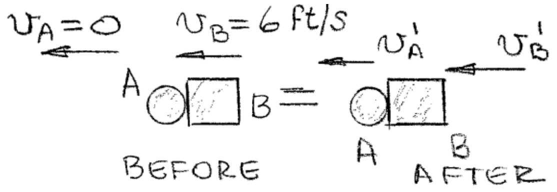
After rebound at C the y component is multiplied by e and the x component is unchanged

$$\overline{v}'' = -ev_x\mathbf{i} + ev_y\mathbf{j} = -e\left(v_x\mathbf{i} - v_y\mathbf{j}\right)$$

so $\overline{v}'' = -e\overline{v}$ And the final velocity is parallel to the original velocity ◀

Chapter 13, Solution 176.

Velocities just after impact



Total momentum in the horizontal direction is conserved

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'; \quad \frac{1.5}{g}(0) + \frac{2.5}{g}(6) = \frac{1.5}{g} v_A' + \frac{2.5}{g} v_B'$$

$$15 = 1.5 v_A' + 2.5 v_B' \qquad (1)$$

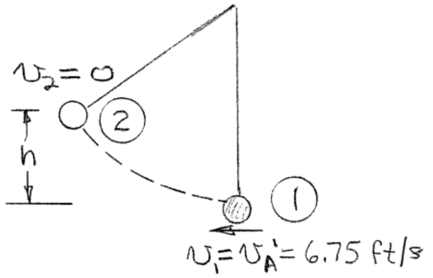
Relative velocities

$$(v_A - v_B)e = v_B' - v_A': \quad (0-6)(0.8) = v_B' - v_A' \Rightarrow v_B' - v_A' = -4.8 \qquad (2)$$

Solving (1) and (2) simultaneously

$$v_A' = 6.75 \text{ ft/s} \qquad v_B' = 1.95 \text{ ft/s}$$

(*a*)



Conservation of energy

$$T_1 = \frac{1}{2} m_A v_A^2 \qquad V_1 = 0$$

$$T_1 = \frac{1}{2} \frac{(1.5 \text{ lb})(6.75 \text{ ft/s})^2}{32.2 \text{ ft/s}^2} = 1.06124$$

continued

$$T_2 = 0$$

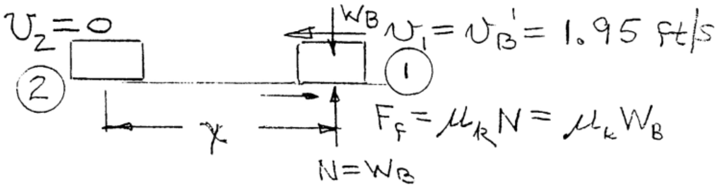
$$V_2 = m_A gh = 1.5h$$

$$T_1 + V_1 = T_2 + V_2: \quad 1.06124 = 1.5h$$

$$h = 0.70749 \text{ ft} = 8.9899 \text{ in.}$$

$$h = 8.49 \text{ in.} ◀$$

(*b*) Work and energy



$$T_2 = 0$$

$$T = \frac{1}{2} m_B v_B'^2 = \frac{1}{2}\left(\frac{2.5}{g}\right)(1.95)^2 = 0.14761$$

$$U_{1-2} = -F_f x = -\mu_k W_B x = -0.6(2.5)x = -1.5x$$

$$T_1 + U_{1-2} = T_2: \quad 0.14761 - 1.5x = 0$$

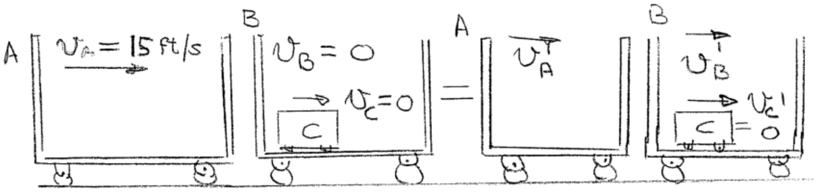
$$x = 0.0984 \text{ ft} = 1.1808 \text{ in.}$$

$$x = 1.181 \text{ in.} ◀$$

Chapter 13, Solution 177.

(*a*) Impact between *A* and *B*

Total momentum conserved



$$m_A = m_B = \frac{80}{g}$$

$$m_C = \frac{30}{g}$$

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \Rightarrow v_A' + v_B' = 15 \qquad (1)$$

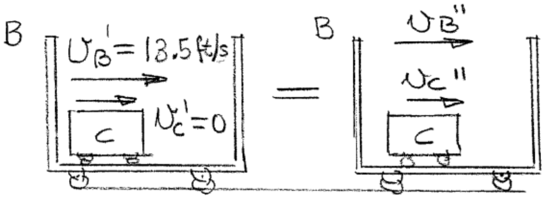
Relative velocities

$$(v_A - v_B)e_{AB} = v_B' - v_A' \Rightarrow (15 - 0)(0.8) = v_B' - v_A' : \quad v_B' - v_A' = 12 \qquad (2)$$

Solving (1) & (2) $\qquad v_B' = 13.5 \text{ ft/s} \longrightarrow$

Impact between *B* and *C* (after *A* hits *B*)

Total momentum conserved



$$\xrightarrow{+} m_B v_B' + m_C v_C' = m_B v_B'' + m_C v_C'' : \quad \frac{80}{g}(13.5) + \frac{30}{g}(0) = \frac{80}{g} v_B'' + \frac{30}{g} v_C''$$

$$1080 = 80 v_B'' + 30 v_C'' \qquad (3)$$

continued

Relative velocities

$$\left(v_B' - v_C'\right)e_{BC} = v_C'' - v_B'': \quad \left(13.5 - 0\right)\left(0.3\right) = v_C'' - v_B''$$

$$4.05 = v_C'' - v_B'' \qquad (4)$$

Solving (3) and (4) $\quad v_B'' = 8.7136 \text{ ft/s} \qquad v_C'' = 12.7636 \text{ ft/s}$

$$v_B'' = 8.71 \text{ ft/s} \blacktriangleleft$$

(*b*)

$$\Delta T_L = \left(T_B' + T_C'\right) - \left(T_B'' + T_C''\right)$$

$$T_B' = \frac{1}{2}m_B\left(v_B'\right)^2 = \frac{1}{2}\left(\frac{80 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(13.5 \text{ ft/s}\right)^2 = 226.39 \text{ lb}\cdot\text{ft}$$

$$T_C' = 0$$

$$T_B'' = \frac{1}{2}m_B\left(v_B''\right)^2 = \frac{1}{2}\left(\frac{80 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(8.7136 \text{ ft/s}\right)^2 = 94.319 \text{ lb}\cdot\text{ft}$$

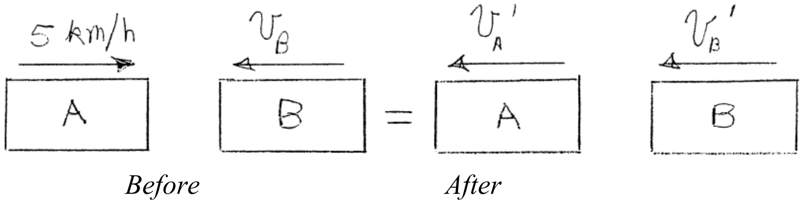
$$T_C'' = \frac{1}{2}m_C\left(v_C''\right)^2 = \frac{1}{2}\left(\frac{30 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(12.764 \text{ ft/s}\right)^2 = 75.894 \text{ lb}\cdot\text{ft}$$

$$\Delta T_L = \left(226.39 + 0\right) - \left(94.319 + 75.894\right) = 56.177$$

$$\Delta T_L = 56.2 \text{ lb}\cdot\text{ft} \blacktriangleleft$$

Chapter 13, Solution 178.

(*a*)


5 km/h
v_B
v_A'
v_B'
A
B
=
A
B
Before
After

$$m_A = m_B = m$$

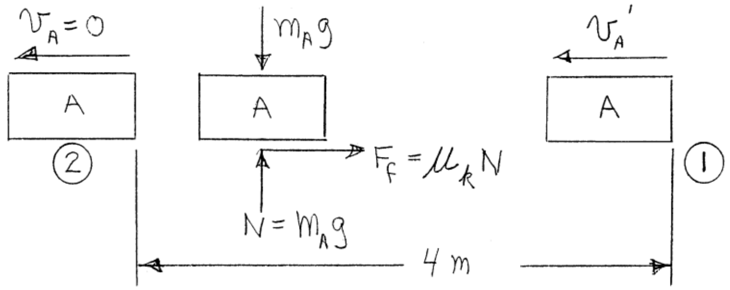
$$5 \text{ km/h} = 1.3889 \text{ m/s}$$

Conservation of total momentum

$$\overset{+}{\longleftarrow} \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$1.3889 - v_B = -v_A' - v_B' \qquad v_A' + v_B' = v_B - 1.3889 \qquad (1)$$

Work and energy – car *A* (after impact)


$v_A = 0$
$m_A g$
v_A'
A
A
A
②
$F_f = \mu_k N$
①
$N = m_A g$
4 m

$$T_1 = \frac{1}{2} m_A v_A'^2$$

$$T_2 = 0$$

$$U_{1-2} = F_f(4)$$

$$= -\mu_k m_A g(4)$$

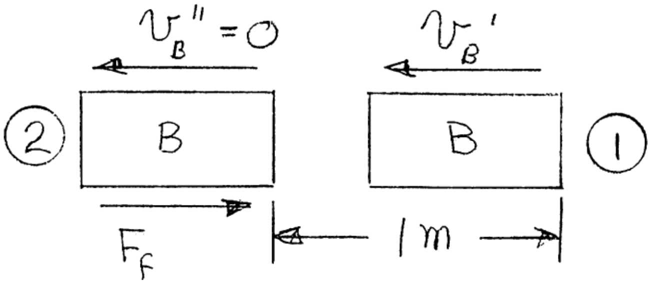
$$T_1 + U_{1-2} = T_2; \quad \frac{1}{2} m_A v_A'^2 - 4\mu_k m_A g = 0$$

continued

$$v_A'^2 = 2(4\text{ m})(0.3)\left(9.81\text{ m/s}^2\right) = 23.544\text{ m}^2/\text{s}^2$$

$$v_A' = 4.852\text{ m/s}$$

Car B – (after impact)



$$T_1 = \frac{1}{2}m_B v_B'^2,\ T_2 = 0$$

$$U_{1-2} = -\mu_k m_B g(1)$$

$$T_1 + U_{1-2} = T_2:\ \ \frac{1}{2}m_B v_B'^2 - \mu_k m_B g(1) = 0$$

$$v_B'^2 = 2(0.3)(1\text{ m})\left(9.81\text{ m/s}^2\right) = 5.886\text{ m}^2/\text{s}^2;\ \ v_B' = 2.426\text{ m/s}$$

From (1) $v_B = v_A' + v_B' + 1.3889 = 4.852 + 2.426 + 1.38$

$v_B = 31.2\text{ km/h}$ ◀

(*b*) Relative velocities

$$\xleftarrow{+}\ (-v_A - v_B)e = v_B' - v_A'$$

$$(-1.3889 - 8.667)e = 2.426 - 4.852$$

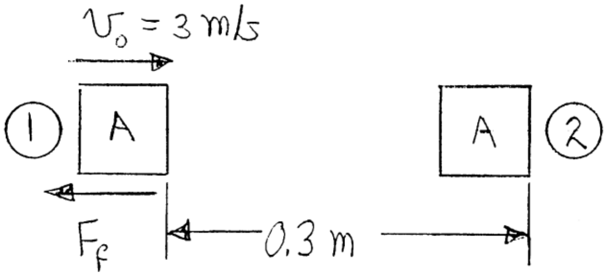
$$(-10.0559)e = -2.426$$

$e = 0.241$ ◀

Chapter 13, Solution 179.

(*a*) Work and energy

Velocity of *A* just before impact with *B*



$$T_1 = \frac{1}{2}m_A v_0^2 \qquad T_2 = \frac{1}{2}m_A\left(v_A^2\right)_2$$

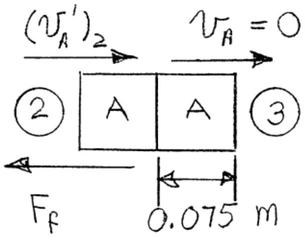
$$U_{1-2} = -\mu_k m_A g\,(0.3\text{ m})$$

$$F_f = \mu_k N = \mu_k m_A g$$

$$T_1 + U_{1-2} = T_2:\quad \frac{1}{2}(0.4\text{ kg})(3\text{ m/s})^2 - 0.3(0.4\text{ kg})\left(9.81\text{ m/s}^2\right)(0.3\text{ m}) = \frac{1}{2}(0.4\text{ kg})\left(v_A^2\right)_2$$

$$\left(v_A^2\right)_2 = 7.2342 \qquad (v_A)_2 = 2.6896\text{ m/s}$$

Velocity of *A* after impact with *B* $(v_A')_2$



$$T_2 = \frac{1}{2}m_A\left(v_A'^2\right)_2 \qquad T_3 = 0$$

$$U_{2-3} = -\mu_k m_A g\,(0.075\text{ m})$$

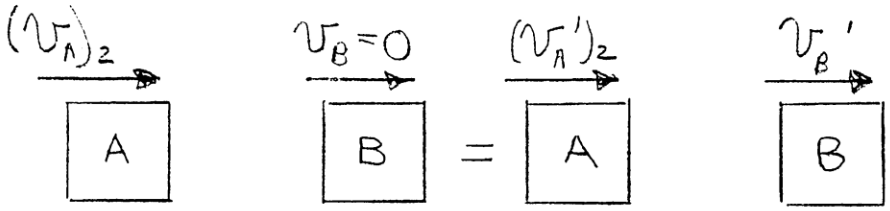
continued

$$T_2 + U_{2-3} = T_3$$

$$\frac{1}{2}(0.4\text{ kg})\left(v_A'^2\right)_2 - 0.3(0.4\text{ kg})\left(9.81\text{ m/s}^2\right)(0.075\text{ m}) = 0$$

$$(v_A')_2 = 0.6644\text{ m/s}$$

Conservation of momentum as *A* hits *B*



$$(v_A)_2 = 2.6896\text{ m/s}$$

$$(v_A')_2 = 0.6644\text{ m/s}$$

$$m_A(v_A)_2 + m_B(v_B) = m_A(v_A')_2 + m_B(v_B') \qquad m_A = m_B$$

$$2.6896 + 0 = 0.6644 + v_B' \qquad v_B' = 2.0252\text{ m/s}$$

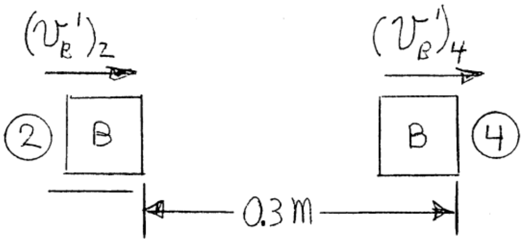
Relative velocities (*A* and *B*)

$$\xrightarrow{+} \left[(v_A)_2 - v_B\right]e_{AB} = v_B' - (v_A')_2$$

$$(2.6896 - 0)e_{AB} = 2.0252 - 0.6644 \qquad e_{AB} = 0.506 \blacktriangleleft$$

Work and energy

Velocity of *B* just before impact with *C*



$$T_2 = \frac{1}{2}m_B(v_B')_2^2 = \frac{0.4}{2}(2.0252)^2 = 0.8203$$

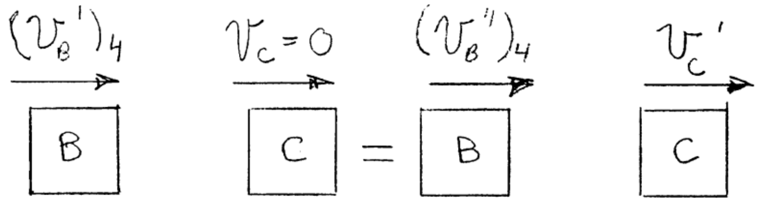
$$T_4 = \frac{1}{2}m_B(v_B')_4^2 = \frac{0.4}{2}(v_B')_4^2$$

$$U_{2-4} = -\mu_k m_B g(0.30) = -0.35316$$

$$T_1 + U_{2-4} = T_4: \quad 0.8203 - 0.35316 = 0.2(v_B')_4^2$$

$$(v_B')_4 = 1.5283\text{ m/s}$$

Conservation of momentum as *B* hits *C*



$$m_C = 1.2 \text{ kg}, \; m_B = 0.4 \text{ kg}$$

$$\xrightarrow{+} \; m_B (v_B')_4 + m_C v_C = m_B (v_B'')_4 + m_C v_C'$$

$$0.4(1.5283) + 0 = 0.4(v_B'')_4 + 1.2 v_C'$$

$$\text{Velocity of } B \text{ after } B \text{ hits } C, \; (v_B'')_4 = 0$$

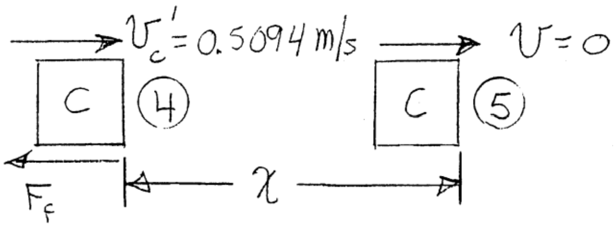
$$\text{With} (v_B'')_4 = 0; \quad 0.61132 = 1.2 v_C' \Rightarrow v_C' = 0.5094 \text{ m/s}$$

Relative velocities (*B* and *C*)

$$\left[(v_B')_4 - v_C\right] e_{BC} = v_C' - (v_B'')_4; \quad (1.5283 - 0) e_{BC} = 0.5094 - 0$$

$$e_{BC} = 0.333 \blacktriangleleft$$

(*b*) Work and energy – Block C



$$T_5 = 0$$

$$T_4 = \frac{1}{2}(1.2)(0.5094)^2 = 0.15569$$

$$U_{4-5} = -\mu_k mgx = -0.3(1.2)(9.81)x = -3.5316x$$

$$T_4 + U_{4-5} = T_5: \quad 0.15569 - 3.5361x = 0 \Rightarrow x = 0.044 \text{ m}$$

$$x = 44.0 \text{ mm} \blacktriangleleft$$

Chapter 13, Solution 180.

Conservation of energy before impact

$$T + V = 0 + 8\ \text{kg}\left(9.81\ \text{m/s}^2\right)\left(0.15\ \text{m}\right) = \frac{1}{2}\left(8\ \text{kg}\right)v_0^2$$

$$v_0 = 1.7155\ \text{m/s}$$

$$\left.\begin{array}{rrc} \text{Cylinder C:} & +\downarrow & 8v_0 - \int F_1\,dt = v_C' \\ \text{Platform A:} & +\downarrow & 0 + \int F_1\,dt - \int F_2\,dt = 5v_A' \\ \text{Counterweight B:} & +\uparrow & 0 + \int F_2\,dt = 5v_A' \\ \text{Restitution:} & & v_A' - v_C' = 0.8v_0 \end{array}\right\} \text{4 unknowns}$$

Simultaneous solution 4 Equations and 4 unknowns

(a) $v_0' = 0,\ v_A' = 1.372\ \text{m/s}\ \downarrow$ ◀

(b) $\int F_2\,dt = 6.86\ \text{N}\cdot\text{s}\ \uparrow$ ◀

Chapter 13, Solution 181.

(*a*) A and B after the first impact

$$\left[\begin{aligned}&T_1 = \frac{1}{2}(5\text{ kg})(1.372\text{ m/s})^2 + \frac{1}{2}(5\text{ kg})(1.372\text{ m/s})^2\\ &T_2 = \frac{1}{2}(5)v_2^2 + \frac{1}{2}(5)v_2^2\\ &U_{1\to 2} = +Td - Td = 0\end{aligned}\right.$$

$$\therefore\ v_2 = v_A$$

$$v_A = 1.372\text{ m/s}\downarrow \blacktriangleleft$$

$$d = v_1 t = 1.372\ t$$

C after the first impact

$$d = 0 + 9.81\left(\frac{t^2}{2}\right),\ \therefore\ t = 0.2798\text{ s}$$

At which time $v_C = 9.81t$ $\qquad v_C = 2.74\text{ m/s}\downarrow \blacktriangleleft$

(*b*) Second impact: (As before)

C: $+\downarrow$ $\quad 8(2.7448) - \int F_1\,dt = 8v_C''$

A: $+\downarrow$ $\quad 5(1.3724) + \int F_1\,dt - \int F_2\,dt = 5v_A''$

B: $+\uparrow$ $\quad 5(1.3724) + \int F_2\,dt = 5v_A''$

Restitution: $\quad v_A'' - v_C'' = 0.8[2.7448 - 1.3724]$

With 4 unknowns and four equations, solve for

$$v_A'' = 2.47\text{ m/s}\downarrow \blacktriangleleft$$

$$v_C'' = 1.372\text{ m/s}\downarrow \blacktriangleleft$$

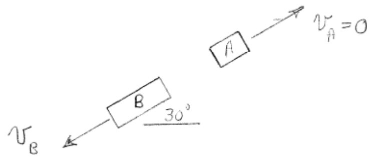
Chapter 13, Solution 182.

Velocity of *A* just before impact, v_0

$$v_0 = 2\sqrt{gh} = \sqrt{2\left(32.2\text{ ft/s}^2\right)\left(3.6\text{ ft}\right)\sin 30^\circ}$$

$$= \sqrt{2(32.2)(3.6)(0.5)} = 10.7666\text{ ft/s}$$

After impact



Conservation of momentum

$$\overset{+}{\nearrow}_{30^\circ}\; m_A v_B = m_B v_B - m_A v_A$$

$$\left(\frac{0.6}{g}\right)v_0 = (1.8g)v_B - \left(\frac{0.6}{g}\right)v_A \qquad (1)$$

g's cancel

Restitution

$$\left(v_A + v_B\right) = e\left(v_0 + 0\right) = 0.9v_0 \qquad (2)$$

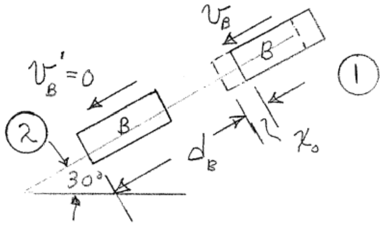
Substituting for v_B from (2) in (1)

$$0.6v_0 = 1.8(0.9v_0 - v_A) - 0.6v_A;\; 2.4v_A = 1.02v_B$$

(*a*) *A* moves up the distance *d* where,

$$\frac{1}{2}m_A v_A^2 = m_A g d \sin 30^\circ;\; \frac{1}{2}(4.5758\text{ ft/s})^2 = (32.2\text{ ft/s}^2)\,d\,(0.5)$$

$$\nearrow_{30^\circ}\; d_A = 0.65025\text{ ft} = 7.80\text{ m} \blacktriangleleft$$



(*b*) Static deflection $= x_0$, *B* moves down $\overset{d_B}{\swarrow}_{30^\circ}$

Conservation of energy (1) to (2)

Position (1) – spring deflected, x_0

$$kx_0 = m_B g \sin 30^\circ$$

$$T_1 + V_1 = T_2 + V_2; \qquad T_1 = \frac{1}{2} m_B v_B^2, \qquad T_2 = 0$$

$$V_1 = V_e + V_g = \frac{1}{2} k x_0^2 + m_B g d_B \sin 30°$$

$$V_2 = V_e' + V_g' = \int_0^{x_0 + d_B} kx dx = \frac{1}{2} k \left(d_B^2 + 2 d_B x_0 + x_0^2 \right)$$

$$\frac{1}{2} k x_0^2 + mg d_B \sin 30° + \frac{1}{2} m_B v_B^2 = \frac{1}{2} k \left(d_B^2 + 2 d_B x_0 + x_0^2 \right) + 0 + 0$$

$$\therefore\ k d_B^2 = m_B v_B^2; \qquad 34 d_B^2 = \left(\frac{1.8}{32.2} \right) (5.1141)^2$$

$$d_B = 0.20737 \text{ ft}$$

$$d_B = 2.49 \text{ in.} \blacktriangleleft$$

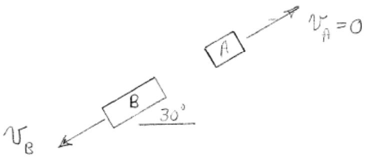
Chapter 13, Solution 183.

Velocity of *A* just before impact, v_0

$$v_0 = \sqrt{2gh} = \sqrt{2\left(32.2\text{ ft/s}^2\right)\left(3.6\text{ ft}\right)\sin 30^\circ}$$

After impact

$$v_0 = \sqrt{2(32.2)(3.6)(0.5)} = 10.7666\text{ ft/s}$$



Conservation of momentum

$$\overset{+}{\swarrow}30^\circ \quad m_A v_0 = m_B v_B - m_A v_A; \quad \left(\frac{0.6}{g}\right)v_0 = \left(\frac{1.8}{g}\right)v_B \qquad (1)$$

g's cancel

Restitution

$$\left(v_A + v_B\right) = e\left(v_0 + 0\right); \quad v_B = ev_0 \qquad (2)$$

From (1) $\quad v_B = \left(\frac{0.6}{1.8}\right)v_0 = \left(\frac{0.6}{1.8}\right)\left(10.7666\text{ ft/s}\right) = 3.5889\text{ ft/s}$

From (2) $\quad e = \left(v_B/v_0\right), \quad e = \frac{1}{3}$

(*a*) $\quad e = 0.333$ ◀

(*b*) Energy loss

$$\Delta\text{ Energy} = m_A g\left(3.6\right)\sin 30^\circ - \frac{1}{2}m_B v_B{}^2$$

$$= \left(0.6\text{ lb}\right)\left(3.6\text{ ft}\right)\left(0.5\right) - \frac{1}{2}\left(\frac{1.8}{32.2}\right)\left(3.5889\text{ ft/s}\right)^2$$

$$= 1.08 - 0.36 = 0.72\text{ ft}\cdot\text{lb}$$

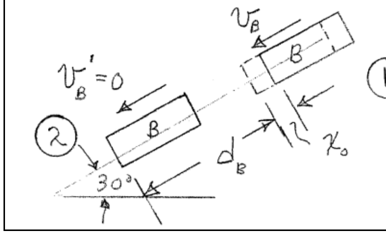
Loss $= 0.720\text{ ft}\cdot\text{lb}$ ◀

(*c*) Static deflection $= x_0$, *B* moves down $\overset{d_B}{\swarrow}30^\circ$

Conservation of energy ① *to* ②

Position ①-spring deflected, x_0

$$kx_0 = m_B g\sin 30^\circ$$



$$T_1 + V_1 = T_2 + V_2; \qquad T_1 = \frac{1}{2}m_B v_B^2, \qquad T_2 = 0$$

$$V_1 = V_e + V_g = \frac{1}{2}kx_0^2 + m_B g d_B \sin 30°$$

$$V_2 = V_e' + V_g' = \int_0^{x_0 + d_B} kx dx = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right)$$

$$\frac{1}{2}kx_0^2 + mgd_B \sin 30° + \frac{1}{2}m_B v_B^2 = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right) + 0 + 0$$

$$\therefore\ kd_B^2 = m_B v_B^2; \qquad 34\,d_B^2 = \left(\frac{1.8}{32.2}\right)(3.5889)^2$$

$$d_B = 0.1455 \text{ ft}$$

$$d_B = 1.746 \text{ in.} \blacktriangleleft$$

Chapter 13, Solution 184.

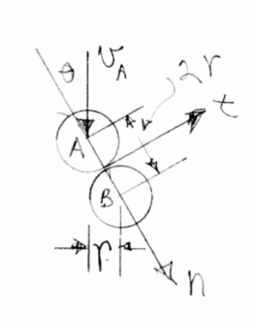
Ball A falls

$$\overset{0}{\cancel{T_1}} + V_1 = T_2 + \overset{0}{\cancel{V_2}} \qquad \text{(Put datum at 2)}$$

$$mgh = \frac{1}{2}mv_A^2 \Rightarrow v_A = \sqrt{2gh}$$

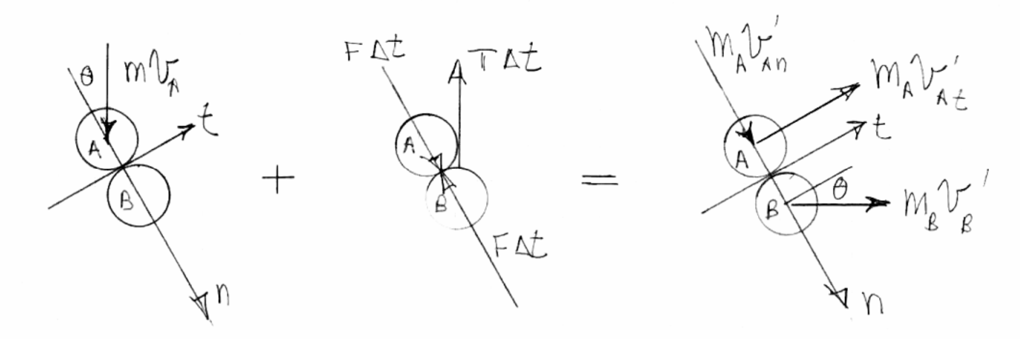
$$= \sqrt{(2)(9.81)(0.2)} = 1.9809 \text{ m/s}$$

Impact



$$\theta = \sin^{-1}\frac{r}{2r} = 30°$$

Impulse–Momentum



Unknowns v'_B, v'_{At}, v'_{An}

x-dir$\longrightarrow$

$$0 + 0 = m_B v'_B + m_A v'_{An}\sin 30° + m_A v'_{At}\cos 30° \tag{1}$$

We need more equations

Coefficient of restitution

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

For our problem

$$v'_B \sin 30° - v'_{An} = e(v_A \cos 30° - 0) \qquad (2)$$

System = *A*

t-dir $\qquad m_A(-v_A \sin 30°) = m_A v'_{At} \qquad (3)$

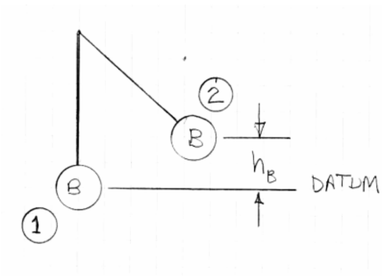
Solve 3 equations and 3 unknowns (maple) using $m_A = m_B = m$

$$v'_B = 1.3724 \text{ m/s} \longrightarrow$$

$$v'_{An} = -1.029 \text{ m/s}$$

$$v'_{At} = -0.9905 \text{ m/s}$$

Now lets look at *B* after impact.



$$T_1 + \cancelto{0}{V_1} = \cancelto{0}{T_2} + V_2$$

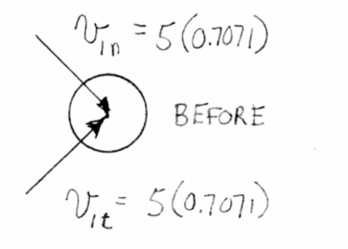
$$\frac{1}{2}m(v'_B)^2 = mgh_B$$

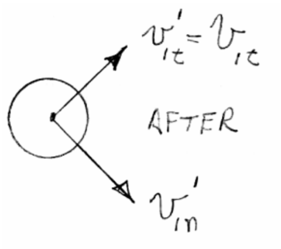
So $\qquad h_B = \dfrac{(v'_B)^2}{2g} = \dfrac{(1.3724)^2}{(2)(9.81)}$

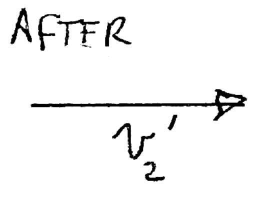
$$= 0.0960 \text{ m}$$

$h_B = 96.0$ mm ◀

Chapter 13, Solution 185.


$v_{1n} = 5(0.7071)$
BEFORE
$v_{1t} = 5(0.7071)$


$v'_{1t} = v_{1t}$
AFTER
v'_{1n}

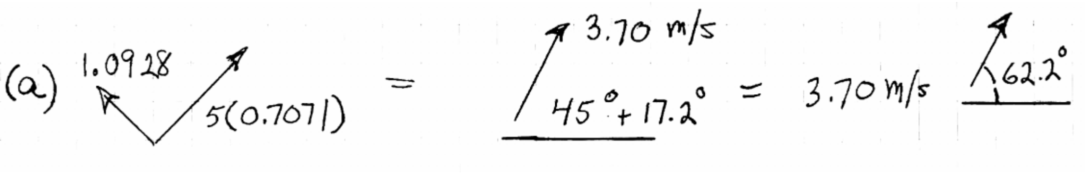


Momentum: $+\searrow \quad mv_{1n} + 0 = mv'_{1n} + Mv'_2(0.7071)$

$$(2\text{ kg})(5\text{ m/s})(0.7071) = 2\text{ kg } v'_{1n} + 9\text{ kg } v'_2(0.7071)$$

Restitution: $v'_2(0.7071) - v'_{1n} = 0.6v_{1n} = 0.6(5)(0.7071)$

Solve for $v'_2 = \left(\frac{16}{11}\right)$ m/s, $v'_{1n} = -\left(\frac{17}{11}\right)(0.7071)$


(a)
1.0928
5(0.7071)
=
3.70 m/s
45° + 17.2°
=
3.70 m/s
62.2°

◀

(b) Conservation of energy – cylinder + spring:

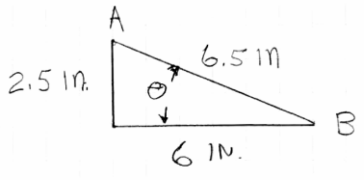
$$\frac{1}{2}kx_0^2 + \frac{1}{2}M(v'_2)^2 = \frac{1}{2}kx_2^2$$

$$\frac{20{,}000}{2}(0.05)^2 + \frac{1}{2}(9)\left(\frac{16}{11}\right)^2 = \frac{20{,}000}{2}x_2^2 = 34.52$$

$x_2 = 0.05875$ m, $\qquad F = kx_2 = 20{,}000\ \frac{\text{N}}{\text{m}}(0.0587\text{ m}) = 1175\text{ N}$ ◀

Chapter 13, Solution 186.

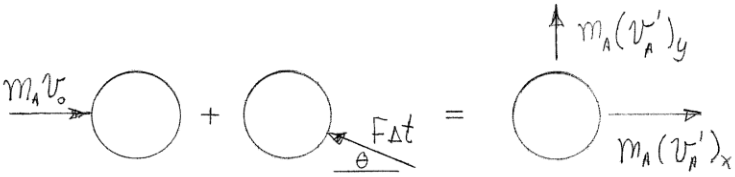
Angle of impulse force from geometry of *A* and *B*


A
B
2.5 in.
6.5 in
6 in.
θ

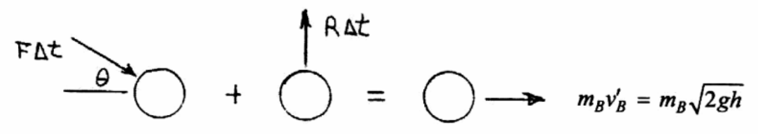
$$\theta = \cos^{-1}\left(\frac{6}{6.5}\right) = 22.62°$$

Total momentum conserved

Ball *A:*

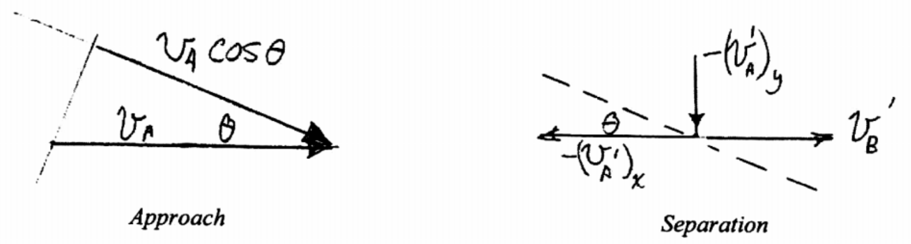

$m_A v_0$
+
$F\Delta t$
θ
=
$m_A(v_A')_y$
$m_A(v_A')_x$

Ball *B:*


$F\Delta t$
θ
+
$R\Delta t$
=
$m_B v_B' = m_B\sqrt{2gh}$

(1)

Restitution


$v_A \cos\theta$
v_A
θ
Approach
$-(v_A')_y$
θ
$-(v_A')_x$
v_B'
Separation

$$e = \frac{v_B'\cos\theta - (v_A')_x\cos\theta + (v_A')_y\sin\theta}{v_A\cos\theta} \qquad v_A = v_0 = 6 \text{ ft/s}$$

$$e = \frac{v_B' - (v_A')_x + (v_A')_y\tan\theta}{v_A} = \frac{v_B' - (v_A')_x + (v_A')_y\left(\frac{25}{6}\right)}{2}$$

continued

$$A:\quad \cancel{m}_A v_A \sin\theta = \cancel{m}_A\left(v'_A\right)_x\left(\sin\theta\right) + \cancel{m}_A\left(v'_A\right)_y\left(\cos\theta\right)$$

$$v_A \tan\theta = (v'_A)_x \tan\theta + (v'_A)_y;\quad 6\left(\frac{2.5}{6}\right) = (v'_A)_x\left(\frac{2.5}{6}\right) + (v'_A)_y$$

$$15 = 2.5\left(v'_A\right)_x + 6\left(v'_A\right)_y \qquad (2)$$

$$A + B:\quad m_A v_A = m_A\left(v'_A\right)_x + m_B v'_B;\quad \left(\frac{50}{g}\right)(6\text{ ft/s}) = \left(\frac{50}{g}\right)(v'_A)_x + \left(\frac{4.6}{g}\right)v'_B \qquad (3)$$

g's cancel

From equation (1) $\quad v'_B = \sqrt{2(32.2\text{ ft/s}^2)(0.75\text{ ft})} = 6.9498\text{ ft/s}$

From equation (3) $\quad (50)(6) = 50(v'_A)_x + 4.6(6.9498)$

$$(v'_A)_x = 5.3606\text{ ft/s}$$

From equation (2) $\quad 15 = 2.5(5.3606) + 6(v'_A)_y$

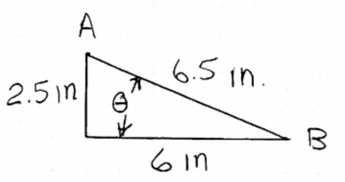
$$(v'_A)_y = 0.2664\text{ ft/s}$$

$$e = \frac{6.9498 - 5.3606 + 0.2664\left(\dfrac{2.5}{6}\right)}{6} = 0.2834$$

$e = 0.283$ ◀

Chapter 13, Solution 187.

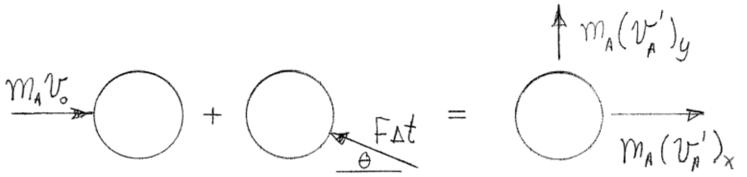
Angle of impulse force from geometry of *A* and *B*


A
2.5 in
6.5 in.
θ
B
6 in

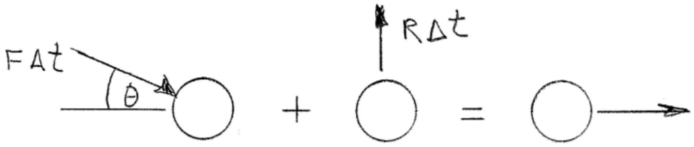
$$\theta = \cos^{-1}\left(\frac{6}{6.5}\right) = 22.62^\circ$$

Momentum consideration

Ball *A:*

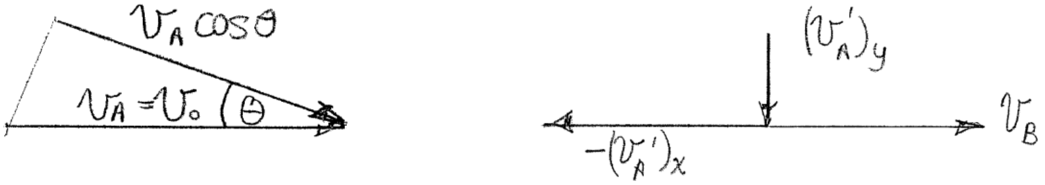

$m_A v_0$
+
$F\Delta t$
θ
=
$m_A(v_A')_y$
$m_A(v_A')_x$

Ball *B:*


$F\Delta t$
θ
+
$R\Delta t$
=

$$m_B v_B' = m_B\sqrt{2gh} \qquad (1)$$

Restitution


$v_A \cos\theta$
$v_A = v_0$
θ
$(v_A')_y$
$-(v_A')_x$
v_B'

Approach *Separation*

$$e = \frac{v_B'\cos\theta - (v_A')_x\cos\theta + (v_A')_y\sin\theta}{v_A\cos\theta} \qquad v_A = v_0 = 6 \text{ ft/s}$$

$$e = \frac{v_B' - (v_A')_x + (v_A')_y\tan\theta}{v_A} = \frac{v_B' - (v_A')_x + (v_A')_y\left(\frac{2.5}{6}\right)}{6}$$

continued

$$A: \quad m_A v_A \sin\theta = m_A\left(v'_A\right)_x\left(\sin\theta\right) + m_A\left(v'_A\right)_y\left(\cos\theta\right)$$

$$v_A \tan\theta = (v'_A)_x \tan\theta + (v'_A)_y; \quad 6\left(\frac{2}{6}\right) = (v'_A)_x\left(\frac{2}{6}\right) + (v'_A)_y$$

$$12 = 2.5\left(v'_A\right)_x + 6\left(v'_A\right)_y \tag{2}$$

$$A + B: \quad m_A v_A = m_A\left(v'_A\right)_x + m_B v'_B; \quad \frac{20}{g}(6) = \frac{20}{g}(v'_A)_x + \frac{2}{g}v'_B$$

$$6 = (v'_A)_x + \frac{v'_B}{10} \qquad (3)$$

From the equation for e

$$e = 0; \quad v'_B - (v'_A)_x + (v'_A)_y\left(\frac{2.5}{6}\right) = 0 \tag{4}$$

$$e = 1; \quad v'_B - (v'_A)_x + (v'_A)_y\left(\frac{2.5}{6}\right) = 6 \tag{5}$$

Simultaneous solution of equations (2), (3) and (4) for $e = 0$ and equations (2), (3) and (5) for $e = 1$ yields

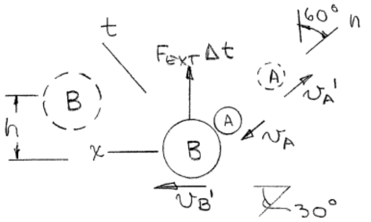
$$e = 0: \quad (v'_A)_x = 5.463 \text{ ft/s}, \quad (v'_A)_y = 0.224 \text{ ft/s}, \quad v'_B = 5.370 \text{ ft/s}$$

$$e = 1: \quad (v'_A)_x = 4.926 \text{ ft/s}, \quad (v'_A)_y = 0.4475 \text{ ft/s}, \quad v'_B = 10.740 \text{ ft/s}$$

$$h = \frac{(v'_B)^2}{2(32.2)} = 0.4478 \text{ ft}, \ 1.791 \text{ ft}$$

$5.37 \text{ in.} \le h \le 21.5 \text{ in.}$ ◀

Chapter 13, Solution 188.

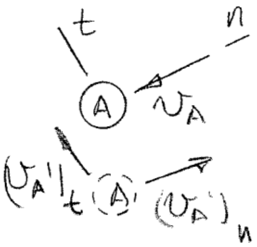


Ball *A* alone

Momentum in *t*-direction conserved

$$m_A(v_A)_t = m_A(v'_A)_t$$

$$(v_A)_t = 0 = (v'_A)_t$$



Thus $$(v'_A)_n = v'_A \measuredangle 60^\circ$$

Total momentum in the *x*-direction is conserved.

$$m_A v_A \sin 60^\circ + m_B(v_B)_x = m_A(-v'_A)\sin 60 + m_B v'_B$$

$$v_A = v_0 = 1.5 \text{ m/s} \qquad (v_B)_x = 0$$

$$0.17(1.5)(\sin 60^\circ) + 0 = -(0.17)(v'_A)(\sin 60^\circ) + (0.34)v'_B$$

$$0.2208 = -0.1472v'_A + 0.34v'_B \qquad (1)$$

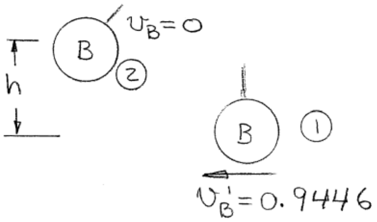
Relative velocity in the *n*-direction

$$\left[-v_A - (v_B)_n\right]e = -v'_B \cos 30^\circ - v'_A;$$

$$(-1.5 - 0)(1) = -0.866v'_B - v'_A \qquad (2)$$

Solving Equations (1) *and* (2) simultaneously

$$v'_B = 0.9446 \text{ m/s}, v'_A = 0.6820 \text{ m/s}$$



Conservation of energy ball *B*

$$T_1 = \frac{1}{2}m_B(v'_B)^2$$

$$T_1 = \frac{1}{2}\frac{W_B}{g}(3.0232)^2 \qquad T_2 = 0$$

continued

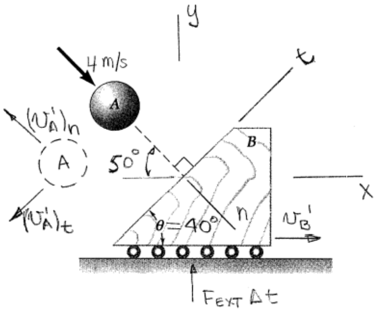
$$V_1 = 0 \qquad V_2 = W_B h$$

$$T_1 + V_1 = T_2 + V_2; \ \frac{1}{2}\frac{\cancel{W_B}}{g}(0.9446)^2 = 0 + \cancel{W_B}h;$$

$$h = \frac{(0.9446)^2}{(2)(9.81)} = 0.0455 \text{ m}$$

$h = 45.5$ mm ◀

Chapter 13, Solution 189.



(*a*) Momentum of the sphere *A* alone is conserved in the *t*-direction.

$$m_A(v_A)_t = m_A(v_A')_t \qquad (v_A)_t = 0$$

$$(v_A')_t = 0 \qquad (v_A')_n = v_A' \measuredangle 50^\circ$$

Total momentum is conserved in the *x*-direction.

$$m_A v_A \cos 50^\circ + m_B v_B = m_A(-v_A')\cos 50^\circ + m_B v_B'$$

$$v_B = 0 \qquad v_A = 4 \text{ m/s}$$

$$2(4)\cos 50^\circ + 0 = 2(-v_A')\cos 50^\circ + 6v_B'$$

$$5.1423 = -1.2855 v_A' + 6v_B' \qquad (1)$$

Relative velocities in the *n*-direction

$$(v_A - v_B)e = (v_B'\cos 50^\circ + v_A'); \;\; v_B = 0, \; v_A = 4 \text{ m/s}$$

$$4(0.5) = 0.6428 v_B' + v_A'; \;\; 2 = 0.6428 v_B' + v_A' \quad (2)$$

Solving Equation (1) and Equation (2) simultaneously

$$v_A' = 1.2736 \text{ m/s}; \;\; v_B' = 1.1299 \text{ m/s}$$

$$v_A' = 1.274 \text{ m/s} \measuredangle 50^\circ \blacktriangleleft$$

$$v_B' = 1.130 \text{ m/s} \longrightarrow \blacktriangleleft$$

(*b*)

$$T_{\text{lost}} = \frac{1}{2}m_A v_A^2 - \frac{1}{2}\left[m_A(v_A')^2 + m_B(v_B')^2\right]$$

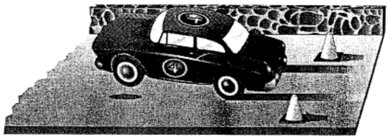
$$= \frac{1}{2}\Big[(2 \text{ kg})(4 \text{ m/s})^2 - (2 \text{ kg})(1.274 \text{ m/s})^2$$

$$- (6 \text{ kg})(1.130 \text{ m/s})^2\Big] = 10.546 \text{ J}$$

$$T_{\text{lost}} = 10.55 \text{ J} \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 190.



First 18 m: Since all the cars' weight is on the rear wheels which skid, the force on the car is

$$F = \mu_k N = (\mu_k)W$$

$$v_{18} = (58 \text{ km/h})(1000 \text{ m/km})\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right)$$

$$= 16.1 \text{ m/s}$$

$$T_1 = 0 \qquad T_2 = \frac{1}{2}mv_{18}^2 = \frac{1}{2}\left(\frac{W}{g}\right)(16.1 \text{ m/s})^2 = (129.6)\frac{W}{g}$$

$$U_{1-2} = (F)(18 \text{ m}) = \mu_k(W)(18 \text{ m})$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 18\mu_k W = (129.6)\left(\frac{W}{g}\right)$$

$$\mu_k = \frac{129.6}{(18)(9.81)} = 0.73395$$

For 400 m: Force moving the car is for the first 18 m,

$$F_1 = (\mu_k)(W) = (0.73395)W$$

For the remaining 382 m, with 75% of weight on rear drive wheels and impending sliding,

$$F_2 = (\mu_s)(0.75)W \qquad \mu_s = \mu_k(0.80) = (0.73395)(0.80) = 0.91744$$

$$F_2 = (0.91744)(0.75)(W) = 0.68808$$

$$T_1 = 0 \qquad T_2 = \frac{1}{2}\left(\frac{W}{g}\right)(v_{400})^2$$

$$U_{1-2} = F_1(18\text{ m}) + F_2(382\text{ m})$$

$$= (0.73395)(W)(18\text{ m}) + (0.68808)(W)(328\text{ m})$$

$$= 13.21W + 262.8W = 276.01W$$

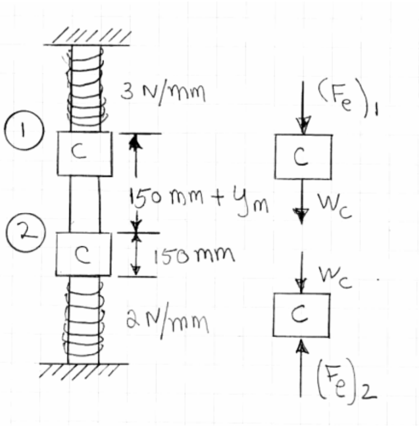
$$T_1 + U_{1-2} = T_2 \qquad 0 + 276.01W = \frac{1}{2}\left(\frac{W}{g}\right)(v_{400})^2$$

$$v_{400}^2 = (2g)276.01 = (2)\left(9.81\text{ m/s}^2\right)(276.01)$$

$$v_{400}^2 = 5415.3 \qquad v_{400} = 73.6\text{ m/s}$$

$$v_{400} = 265\text{ km/h} \blacktriangleleft$$

Chapter 13, Solution 191.



(*a*) Spring constants $\qquad 3\text{ N/mm} = 3000\text{ N/m}$

$$2\text{ N/mm} = 2000\text{ N/m}$$

Max deflection at 2 when velocity of $C = 0$

$$v_1 = 0,\quad T_1 = 0,\quad v_2 = 0,\quad T_2 = 0$$

$$U_{1-2} = U_e + U_g$$

$$U_{1-2} = \int_0^{0.05} (F_e)_1\,dx - \int_0^{y_m} (F_e)_2\,dx + W_C(0.15 + y_m)$$

$$U_{1-2} = \frac{(3000\text{ N/m})}{2}(0.05\text{ m})^2 - \frac{(2000\text{ N/m})}{2}(y_m)^2$$

$$+(3\text{ kg})(9.81\text{ m/s}^2)(0.15 + y_m)$$

$$= 3.750 - 1000(y_m)^2 + 4.4145 + 29.43(y_m)$$

$$T_1 + U_{1-2} = T_2:\quad 0 - 1000(y_m)^2 + 29.43(y_m) + 8.1645 = 0$$

$$y_m = 0.10626\text{ m} \qquad y_m = 106.3\text{ mm} ◀$$

(*b*) Maximum velocity occurs as the lower spring is compressed a distance y'

$$T_1 = 0;\qquad T_2 = \frac{1}{2}m_C v^2 = \frac{1}{2}(3\text{ kg})v^2 = (1.5)v^2$$

$$T_1 + U_{1-2} = T_2;\qquad 0 - (1000)(y')^2 + 29.43(y') + 8.1645 = (1.5)v^2$$

Substitute $\qquad y' = 0.014715\text{ m}$

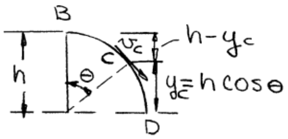
$$\frac{dv^2}{dy'} = 0 \qquad -2000(y') + 29.43 = 0;\qquad y' = 0.014715\text{ m}$$

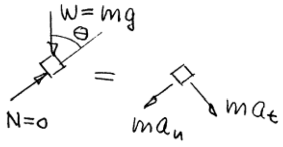
$$-0.21653 + 0.43306 + 8.1645 = 1.5v^2$$

$$v^2 = 5.5873\text{ m/s} \qquad v = 2.36\text{ m/s} ◀$$

Chapter 13, Solution 192.

(*a*)





Block leaves surface at *C* when the normal force $N = 0$

$$\overset{+}{\swarrow}\ mg\cos\theta = ma_n$$

$$g\cos\theta = \frac{v_C^2}{h} \qquad (1)$$

$$v_C^2 = gh\cos\theta = gy$$

Work-energy principle

$$T_B = \frac{1}{2}mv_C^2 \qquad U_{B-C} = W\left(h-y\right) = mg\left(h-y_C\right)$$

$$T_B + U_{B-C} = T_C$$

Use Equation (1)
$$4.5m + mg\left(h-y\right) = \frac{1}{2}mv_C^2$$

$$4.5 + g\left(h-y\right) = \frac{1}{2}gy_C \qquad (2)$$

$$4.5 + gh = \frac{3}{2}gy_C$$

$$y_C = \frac{\left(4.5+gh\right)}{\left(\frac{3}{2}g\right)}$$

$$y = \frac{\left(4.5+\left(9.81\right)\left(1\right)^2\right)}{\frac{3}{2}\left(9.81\right)}$$

$$y = 0.97248\text{ m} \qquad (3)$$

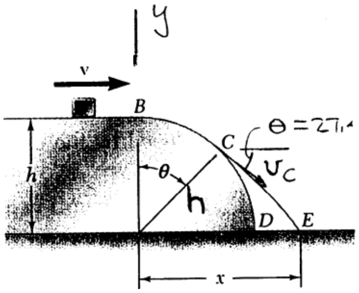
continued

$$y_C = h\cos\theta \qquad \cos\theta = \frac{y_C}{h} = \frac{0.97248}{1\text{ m}} = 0.97248$$

$$\theta = \cos^{-1} 0.97248 = 13.473°$$

$\theta = 13.47°$ ◀

(*b*)



From Equations (1) and (3)

$$v_C = \sqrt{gy} = \sqrt{9.81(0.97248)} = 3.0887 \text{ m/s}$$

At *C*;

$$(v_C)_x = v_C\cos\theta = 3.0887\cos 13.47°$$

$$= 3.0037 \text{ m/s}$$

$$(v_C)_y = -v_C\sin\theta = 3.0887\sin 13.47°$$

$$= -0.71947 \text{ m/s}$$

$$y = y_C + (v_C)_y t - \frac{1}{2}gt^2 = 0.97248 - 0.71947t - \frac{1}{2}(9.81)t^2$$

At *E*:

$$y_E = 0: \quad 4.905t^2 + 0.7194t - 0.97248 = 0$$

$$t = 0.37793 \text{ s}$$

At *E*:

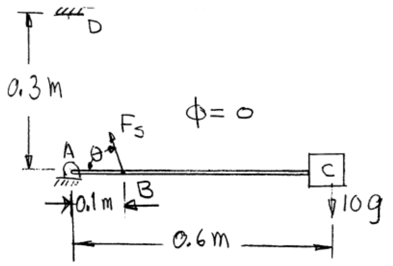
$$x = h\cos\theta + (v_C)_x t = 1(\sin 13.47°) + 3.0037(0.37793)$$

$$= 0.23294 + 1.3519 = 1.3681 \text{ m}$$

$x = 1.368$ m ◀

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 193.



Find unstretched length of the spring

$$\theta = \tan^{-1}\left(\frac{0.3}{0.1}\right) \qquad \theta = 71.565°$$

$$L_{BD} = \sqrt{(0.3)^2 + (0.1)^2} = 0.3162 \text{ m}$$

$$= \text{length at equilibrium}$$

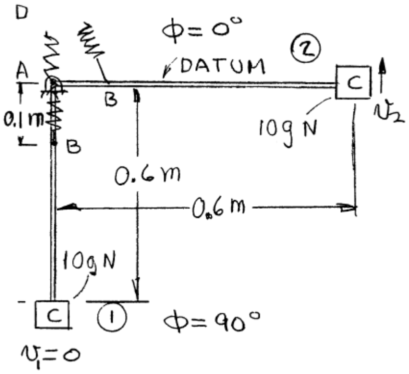
Equilibrium: $$\circlearrowleft + \Sigma M_A = 0.1F_s \sin\theta - 0.6(10 \text{ g}) = 0$$

$$F_s = 63.25 \text{ g}$$

$$F_s = k\Delta L_{BD}: \quad 63.25 \text{ g} = (8000 \text{ N/m})(\Delta L_{BD}) \Rightarrow \Delta L_{BD} = 0.07756 \text{ m}$$

Unstretched length $$L_0 = L_{BD} - \Delta L_{BD} = 0.3162 - 0.07756$$

$$= 0.23864 \text{ m}$$



Spring elongation, $\Delta L'_{BD}$ when $\phi = 90°$

$$\Delta L'_{BD} = (0.3 \text{ m} + 0.1 \text{ m}) - L_0 = 0.4 - 0.23864$$

$$= 0.16136 \text{ m}$$

At ① $$\phi = 90° \qquad V_1 = 0, \qquad T_1 = 0$$

$$V_1 = (V_1)_e + (V_1)_g$$

$$(V_1)_e = \frac{1}{2}k(\Delta L'_{BD})^2 = \frac{8000}{2}(0.16136)^2$$

$$= 104.15 \text{ N}\cdot\text{m}$$

continued

$$(V_1)_g = -10\,\mathrm{g}(0.6) = -58.86\ \mathrm{N\cdot m}$$

$$V_1 = 104.15 - 58.86 = 45.29\ \mathrm{N\cdot m}$$

At ② $\qquad \phi = 0 \qquad (V_2)_e = \frac{1}{2}k(\Delta L_{BD})^2 = \left(\frac{8000}{2}\ \mathrm{N/m}\right)(0.07756\ \mathrm{m})^2$

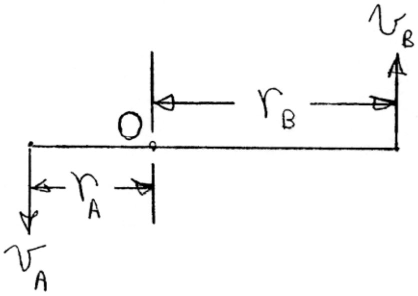
$$= 24.06\ \mathrm{N\cdot m}$$

$$T_2 = \frac{1}{2}mv_2^2 = \left(\frac{10\ \mathrm{kg}}{2}\right)v_2^2 = 5v_2^2$$

$$T_1 + V_1 = T_2 + V_2:\quad 0 + 45.29 = 5v_2^2 + 24.06$$

$$v_2^2 = 4.246 \qquad\qquad v_2 = 2.06\ \mathrm{m/s}$$ ◀

Chapter 13, Solution 194.



$$r_A = 3960 + 1500 = 5460 \text{ mi} = 28.829 \times 10^6 \text{ ft}$$

$$r_B = 3960 + 6000 = 9960 \text{ mi} = 52.589 \times 10^6 \text{ ft}$$

Conservation of momentum $\qquad r_A m v_A = r_B m v_B$

$$v_B = \left(\frac{r_A}{r_B}\right) v_A = \frac{28.829}{52.589} v_A = 0.54819 v_A \qquad (1)$$

Conservation of energy

$$T_A = \frac{1}{2} m v_A^2, \qquad V_A = -\frac{GMm}{r_A}, \qquad T_B = \frac{1}{2} m v_B^2, \qquad V_B = -\frac{GMm}{r_B}$$

$$GM = gR^2 = \left(32.2 \text{ ft/s}^2\right)\left(3960 \text{ mi} \times 5280 \text{ ft/mi}\right)^2$$

$$= 140.77 \times 10^{14} \text{ ft}^3/\text{s}^2$$

$$V_A = -\frac{140.77 \times 10^{14} m}{28.829 \times 10^6} = -488.29 \times 10^6 \text{m}$$

$$V_B = \frac{-140.77 \times 10^{14} m}{52.589 \times 10^6} = -267.68 \times 10^6 \text{m}$$

$$T_A + T_B = T_B + V_B: \ \frac{1}{2} m v_A^2 - 488.29 \times 10^6 \text{m}$$

$$= \frac{1}{2} m v_B^2 - 267.68 \times 10^6 \text{m}$$

$$\frac{1}{2} v_A^2 - 220.61 \times 10^6 = \frac{1}{2} v_B^2$$

Using (1) $\qquad \frac{1}{2} v_A^2 - 220.61 \times 10^6 = \frac{1}{2}\left(0.54819 v_A\right)^2$

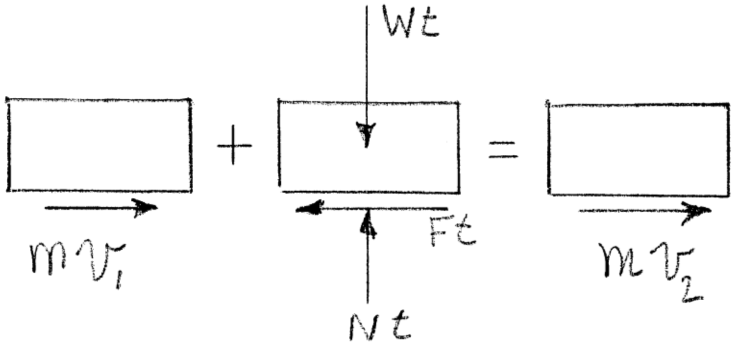
$$0.34974 v_A^2 = 220.61 \times 10^6$$

$$v_A = 25115.39$$

$$v_A = 25.1 \times 10^3 \text{ ft/s} \blacktriangleleft$$

Vector Mechanics for Engineers: Statics and Dynamics, 8/e, Ferdinand P. Beer, E. Russell Johnston, Jr.,
Elliot R. Eisenberg, William E. Clausen, David Mazurek, Phillip J. Cornwell

Chapter 13, Solution 195.



$$Ft = \mu_s Nt = \mu_s mgt$$

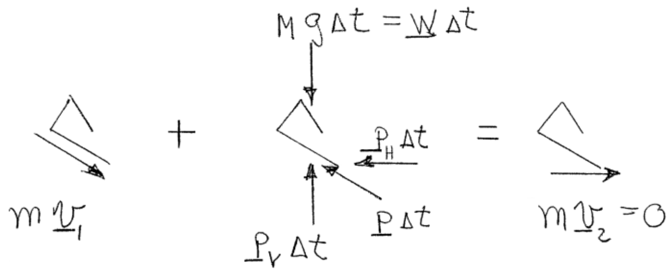
$$v_1 = 60 \text{ mi/h} = 88 \text{ ft/s} \qquad v_2 = 20 \text{ mi/h} = 29.333 \text{ ft/s}$$

$$\cancel{m}v_1 - \mu_s \cancel{m}gt = \cancel{m}v_2 \qquad \mu_s = 0.65$$

$$88 - 0.65(32.2)t = 29.333 \Rightarrow t = 2.803$$

$$t = 2.80 \text{ s} \blacktriangleleft$$

Chapter 13, Solution 196.



$$\Delta t = 0.22$$

$$mv_1 + (P - W)\Delta t = mv_2$$

Horizontal components

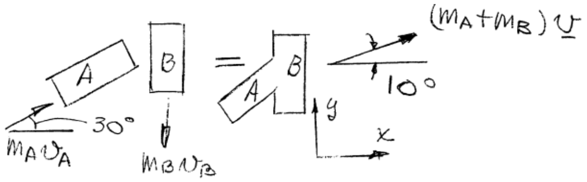
$$\xrightarrow{+} 84(9.14\cos 35^\circ) - P_H(0.22) = 0$$

$$P_H = 2858.69\ \text{kg}\cdot\text{m/s}^2$$

$$P_H = 2.86\ \text{kN} \blacktriangleleft$$

Chapter 13, Solution 197.

(*a*) Total momentum of the two cars is conserved.



$$\Sigma mv,\ \ x:\quad m_Av_A\cos 30° = \left(m_A + m_A\right)v\cos 10° \tag{1}$$

$$\Sigma mv,\ \ y:\quad m_Av_A\sin 30° - m_Bv_B = \left(m_A + m_B\right)v\sin 10° \tag{2}$$

Dividing (1) into (2)

$$\frac{\sin 30°}{\cos 30°} - \frac{m_Bv_B}{m_Av_A\cos 30°} = \frac{\sin 10°}{\cos 10°}$$

$$\frac{v_B}{v_A} = \frac{\left(\tan 30° - \tan 10°\right)\left(m_A\cos 30°\right)}{m_B}$$

$$\frac{v_B}{v_A} = 0.3473\frac{m_A}{m_B} \qquad m_A = \frac{3600}{g} \qquad m_B = \frac{2800}{g}$$

$$\frac{m_A}{m_B} = \frac{3600}{2800} = 1.2857$$

$$v_B = \left(0.3473\right)\left(1.2857\right) = 0.4465v_A$$

Car *A* was going faster ◀

(*b*) Since *B* was the slower car, $v_B = 30$ mi/h

$$v_A = 2.2396v_B$$

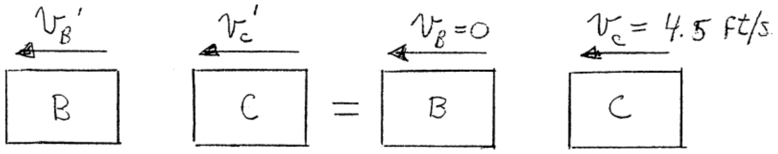
$v_A = 67.2$ mi/h ◀

Chapter 13, Solution 198.

$$m_A = m_B = m_C = m$$

Collision between *B* and *C*

The total momentum is conserved.



$$\xleftarrow{+} \quad mv'_B + mv'_C = mv_B + mv_C \Rightarrow v'_B + v'_C = 4.5 \qquad (1)$$

Relative velocities

$$v'_C - v'_B = e\left(v_B - v_C\right) = 0.8\left(-4.5\right) \Rightarrow v'_B - v'_C = 3.6 \; (2)$$

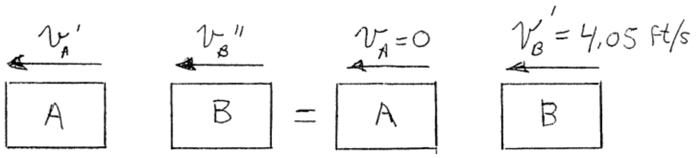
Solving (1) and (2) simultaneously

$v'_B = 4.05$ ft/s ← ◀

$v'_C = 0.450$ ft/s ← ◀

Since $v'_B > v'_C$, Car *B* collides with Car *A*

Collision between *A* and *B*



$$v_A' + v_B'' = 4.05 \qquad (3)$$

Relative velocities

$$\left(v_A - v'_B\right)e = v''_B - v'_A \Rightarrow v''_B - v'_A = 0.5\left(-4.05\right); \; v'_A - v''_B = 2.025 \qquad (4)$$

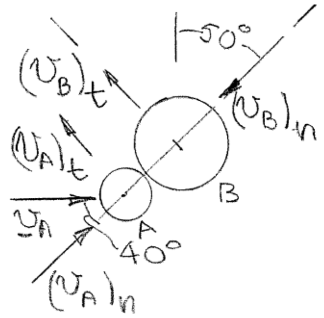
Solving (3) and (4) simultaneously

$v''_B = 1.013$ ft/s ← ◀

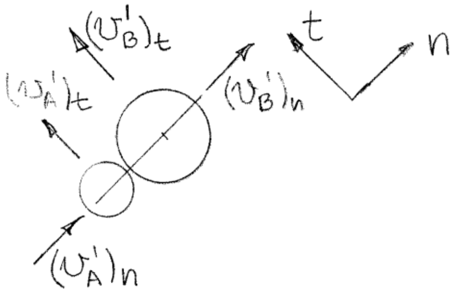
$v'_A = 3.04$ ft/s ← ◀

$$v'_C < v''_B < v'_A \Rightarrow \text{No more collisions}$$

Chapter 13, Solution 199.



Before



After

$$v_A = 18 \text{ ft/s}; \ (v_A)_n = 18\cos 40° = 13.79 \text{ ft/s}; \ (v_A)_t = -18\sin 40° = -11.57 \text{ ft/s}$$

$$v_B = (v_B)_n = -12 \text{ ft/s}; \ (v_B)_t = 0$$

t-direction

Total momentum conserved

$$m_A(v_A)_t + m_B(v_B)_t = m_A(v'_A)_t + m_B(v'_B)_t$$

$$\frac{(1.5 \text{ lb})}{g}(-11.57 \text{ ft/s}) + 0 = \frac{(1.5 \text{ lb})}{g}(v'_A)_t + \frac{(2.5 \text{ lb})}{g}(v'_A)_t$$

$$-17.36 = 1.5(v'_A)_t + 2.5(v'_B)_t \tag{1}$$

Ball *A* alone momentum conserved

$$m_A(v_A)_t = m_A(v'_A)_t \Rightarrow (v'_A)_t = -11.57 \text{ ft/s} \tag{2}$$

Replace $(v'_A)_t$ in (2) in equation (1)

$$-17.36 = 1.5(-11.57) + 2.5(v'_B)_t; \ (v'_B)_t = 0$$

n-Direction Relative velocities

$$\left[(v_A)_n - (v_B)_n\right]e = (v'_B) - (v'_A)_n$$

$$\left[13.79 - (-12)\right]0.8 = (v'_B)_n - (v'_A)_n$$

$$(v'_B)_n - (v'_A)_n = 20.632 \qquad (3)$$

Total momentum conserved

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$\frac{(1.5\text{ lb})}{g}(13.79\text{ ft/s}) + \frac{(2.5\text{ lb})}{g}(-12\text{ ft/s}) = \frac{(1.5\text{ lb})}{g}(v'_A)_n + \frac{(2.5\text{ lb})}{g}(v'_B)_n$$

$$1.5(v'_A)_n + 2.5(v'_B)_n = -9.315 \qquad (4)$$

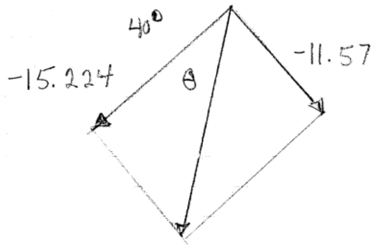
Solve (3) and (4):

$$4(v'_B)_n = 21.633$$

$$(v'_B)_n = 5.408\text{ ft/s}$$

$$(v'_A)_n = -15.224\text{ ft/s}$$

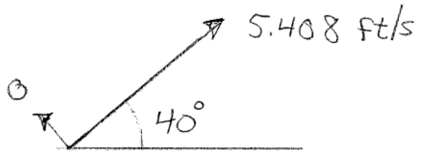
A



$$v_A = \sqrt{(15.224)^2 + (11.57)^2} = 19.12\text{ ft/s},\ \theta = 37.23^\circ$$

$$v_A = 19.12\text{ ft/s} \measuredangle 72.2^\circ \blacktriangleleft$$

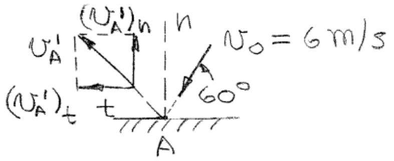
B



$$v_B = 5.41\text{ ft/s} \measuredangle 40^\circ \blacktriangleleft$$

Chapter 13, Solution 200.

(*a*) Rebound at *A*



Conservation of momentum – *t* direction

$$mv_0 \cos 60° = m(v_A')_t \Rightarrow (v_A')_t = 6\cos\theta$$

$$(v_A')_t = 3 \text{ m/s}$$

Coefficient of restitution in the *n* direction

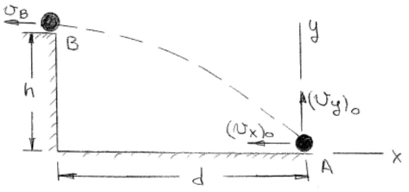
$$\left(-(v_A)_n - 0\right)e = 0 - (v_A')_n: \quad (6\sin 60°)(0.6) = (v_A')_n$$

$$(v_A')_n = 3.12 \text{ m/s}$$

After rebound $\qquad (v_x)_0 = -(v_A')_t = -3 \text{ m/s}$

$$(v_y)_0 = (v_A')_n = 3.12 \text{ m/s}$$

Projectile motion between *A* and *B*



$$x = (v_x)_0 t = -3t, \; v_x = -3 \text{ m/s}$$

$$y = (v_y)_0 t - \frac{1}{2}gt^2 = 3.12t - 4.905t^2;$$

$$v_y = (v_y)_0 - gt = 3.12 - 9.81t$$

At B, $\qquad v_y = 0: \quad 3.12 - 9.81t_{A-B} = 0 \Rightarrow t_{A-B} = 0.318 \text{ s}$

$$y_B = h: \quad h = 3.12t_{A-B} - 4.905t_{A-B}^2 = 0.496 \text{ m}$$

$$x_B = -d = -3t_{A-B}$$

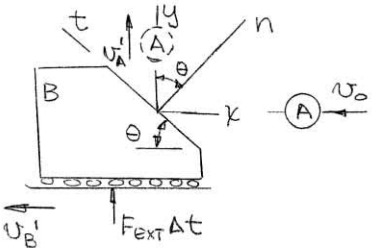
$h = 0.496$ m ◀

$d = 0.953$ m ◀

(*b*) $\qquad v_B = (v_x)_0 = -3 \text{ m/s}$

$v_B = 3.00$ m/s ← ◀

Chapter 13, Solution 201.



(*a*) Momentum of sphere *A* alone is conserved in the *t*-direction.

$$m_A v_0 \cos\theta = m_A v_A' \sin\theta$$

$$v_0 = v_A' \tan\theta \qquad (1)$$

Total momentum is conserved in the *x*-direction.

$$m_B v_B + m_A v_0 = m_B v_B' + m_A \left(v_A'\right)_x \qquad v_B = 0,\ \left(v_A'\right)_x = 0$$

$$0 + \frac{1.5}{g} v_0 = \frac{4.5}{g} v_B' + 0$$

$$v_B' = \frac{v_0}{3} \qquad (2)$$

Relative velocities in the *n*-direction

$$\left(-v_0 \sin\theta - 0\right) e = -v_B' \sin\theta - v_A' \cos\theta$$

$$\left(v_0\right)\left(0.6\right) = v_B' + v_A' \cot\theta \qquad (3)$$

Substituting v_B' from (2) into (3)

$$0.6 v_0 = 0.333 v_0 + v_A' \cot\theta$$

$$0.267 v_0 = v_A' \cot\theta \qquad (4)$$

Divide (4) into (1)

$$\frac{1}{0.267} = \frac{\tan\theta}{\cot\theta} = \tan^2\theta$$

$$\tan\theta = 1.935 \qquad \theta = 62.7^\circ \blacktriangleleft$$

(*b*) From (1) $\qquad v_0 = v_A' \tan\theta = v_A'\left(1.935\right)$

$$v_A' = 0.5168 v_0,\ \ v_B' = \frac{v_0}{3} \qquad (2)$$

continued

$$T_{\text{lost}} = \frac{1}{2}m_A v_A^2 - \frac{1}{2}\left(m_A\left(v_A'\right)^2 + m_B v_B^2\right)$$

$$T_{\text{lost}} = \frac{1}{2}\left(\frac{1.5}{g}\right)\left(v_0\right)^2 - \frac{1}{2}\left[\frac{1.5}{2}\left(0.5168v_0\right)^2 + \frac{4.5}{g}\left(\frac{v_0}{3}\right)^2\right]$$

$$= \frac{v_0^2}{2g}\left[1.5 - 0.40 - 0.50\right] = \frac{0.3v_0^2}{g}$$

$$T_{\text{lost}} = 0.00932v_0^2 \text{ ft}\cdot\text{lb} \blacktriangleleft$$

(For v_0 in ft/s).